

# System Identification of Concrete Arch Dam Using Frequency Domain and Time-Frequency Domain Methods

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## Abstract

System identification is an effective and important method in health monitoring of structures especially in long term behaviors. It looks after any changes such as cracking or abutment instability in the behavior of structure. There are several methods of system identification which investigate dynamic parameters of the structure and each of them has advantages and disadvantages. The results in system identification process are reliable when it is done by several records and similar results are achieved. In this article, dynamic parameters of double curved Karun III dam was identified by different methods and different records. Peak Picking (PP), Frequency Domain Decomposition (FDD) and Wavelet Transform (WT) methods are used to identify the natural frequencies and extracting damping ratios of the Karun III dam. The similarity of the results for different records and different methods proves that the results are reliable.

**Keywords:** System identification, Wavelet transform, Frequency domain decomposition, Karun III.

## 1. INTRODUCTION

Operational modal analysis (OMA) is one of the best methods for health monitoring and detecting of probable damages in structures. OMA methods are based on only-output data [1]. Classical modal parameter identifications are usually based on frequency response functions or impulse response function that require measurement of both the input force and the resulting response. However, for some practical reasons, modal parameters must be extracted only from response data sometimes. In addition, ambient vibration testing (AVT) has advantages over forced vibration and free vibration methods such as low cost, multiplicity and variety of excitations and the continuous use of structure in real condition.

Several techniques are available for OMA such as pick picking (PP) from power spectral density (PSD) functions, least-square curve fitting technique, natural excitation technique (NExT), autoregressive moving average (ARMA), stochastic subspace identification (SSI), frequency domain decomposition (FDD) method and continuous wavelet transform (CWT) method. Frequency Domain Decomposition (FDD) method transforms signal to frequency domain and selects system frequencies during processes [2-4]. continuous Wavelet Transform (CWT) is a time-frequency domain method [5,6]. The advantage of the CWT than other time-frequency method is variability of time and frequency resolutions which made CWT to a multi resolution method. In this paper, PP, CWT and FDD methods were used to calculating frequencies and damping ratios of the recorded motions of the Karun III dam.

## 2. SYSTEM IDENTIFICATION

System identification is used to obtain modal characteristics of structures to observe any changes. OMA are output based methods which extract modal parameters based on dam responses regardless to input signal. Different methods were used to this purpose such as Pick Picking (PP), FDD and continuous Wavelet Transform (CWT). Here, system identification was used to identify modal parameters and dynamic behavior of the Karun III dam. PP, CWT and FDD methods were used as Operational Modal Analysis (OMA) method to calculating modal parameters. PP is a simple method based on power spectral density spectrum. FDD and CWT methods are more sophisticated and have special advantages. CWT is a multi-resolution time-frequency method that separate frequencies precisely. In the first step, the wavelet parameters should be selected in an optimization process. Then frequencies of the record are calculated. After that, the original signal is decomposed to single frequency signals that contain identified frequencies. The calculated single frequency signals do not have any modes interference so

they are appropriate to apply half power method to calculate damping ratios. So, the main advantage of CWT method is its ability to decompose signals. FDD method is a frequency domain method which decomposes power spectral density spectrum to its singular values. The peaks in first singular values shows frequencies of the record and second singular value's peaks are used to verify the weak peaks which appears at the first singular values. The main advantage of the FDD method is its accuracy to decompose signals and to distinguish frequencies.

## 2.1. CONTINUOUS WAVELET TRANSFORM (CWT) METHOD

CWT is a mathematical transform which can analyze signals in both time and frequency domain with variable resolution. The main equation of CWT is:

$$CWT_x^\psi(a, b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} x(t) \bar{\psi}\left(\frac{t-b}{a}\right) dt \quad (1)$$

Where  $b$  and  $a$  are translation and scale parameters respectively.  $x$  is signal and  $t$  indicates that the signal is in time domain and  $\psi$  is wavelet function. In this research the modified Morlet wavelet was used as wavelet function. Mathematical representation of modified Morlet wavelet is:

$$\psi(t) = \frac{1}{\sqrt{\pi f_b}} e^{i2\pi f_c t} e^{-t^2/f_b} \quad (2)$$

Where  $f_b$  and  $f_c$  are band pass and central frequency of wavelet respectively.

As mentioned above Wavelet transform is a multi-resolution transform that resolutions are depend on wavelet parameters. Wavelet time and frequency resolutions are represented as:

$$\Delta t_i = \frac{f_c \sqrt{f_b}}{f_i^2} \quad (3)$$

$$\Delta f_i = \frac{f_i}{f_c 2\pi \sqrt{f_b}} \quad (4)$$

As is clear by changing the value of  $f_b$  and  $f_c$ , different resolutions can be obtained. So these parameters should be optimized to obtain better results. Here, a trial and error process was applied to optimize wavelet parameters. The requested domain for wavelet parameters should satisfy Equation 5 [7].

$$\sqrt{f_b} f_c = (2\alpha) \frac{f_{i+1}}{2\pi \Delta f_{i+1}} \quad (5)$$

Where  $\alpha$  is the parameter defining the overlap between the adjacent Gaussian windows of the modified Morlet wavelet. Kijewski and Kareem suggested the empirical value  $\alpha=2$  which is generally sufficient to distinguish two adjacent frequency components [8].

## 2.2. FREQUENCY DOMAIN DECOMPOSITION (FDD) METHOD

In this method, Power Spectral Density (PSD) matrix of the response signal is calculated according to equation 1 [7].

$$G_{yy}(j\omega) = \bar{H}(j\omega) G_{xx}(j\omega) H(j\omega)^T \quad (6)$$

Where  $G_{xx}$  and  $G_{yy}$  are the PSD of input and output signal respectively and  $H$  is the matrix of frequency response function. The frequency response matrix can be written as:

$$H(j\omega) = \sum_{k=1}^n \frac{Q_k}{j\omega - \lambda_k} + \frac{\bar{Q}_k}{j\omega - \bar{\lambda}_k} \quad (7)$$

Where  $n$  is the number of modes,  $\lambda_k$  is the pole and  $Q_k$  is the residue. By combining equation 1 and 2, the relationship between PSD matrix of the input and output is derived as:

$$G_{yy}(j\omega) = \left[ \sum_{k=1}^n \frac{Q_k}{j\omega - \lambda_k} + \frac{\bar{Q}_k}{j\omega - \bar{\lambda}_k} \right] \cdot G_{xx}(j\omega) \cdot \left[ \sum_{s=1}^n \frac{Q_s}{j\omega - \lambda_s} + \frac{\bar{Q}_s}{j\omega - \bar{\lambda}_s} \right] \quad (8)$$

Assuming that the input is white noise, i.e. its PSD is a constant matrix, after some mathematical calculations the following equation can be obtained:

$$G_{yy}(j\omega) = \sum_{k=1}^n \frac{A_k}{j\omega - \lambda_k} + \frac{\bar{A}_k}{j\omega - \bar{\lambda}_k} + \frac{B_k}{-j\omega - \lambda_k} + \frac{\bar{B}_k}{-j\omega - \bar{\lambda}_k} \quad (9)$$

Where  $A_k$  is the  $k$ th residue matrix of the output PSD. As for the output PSD itself, the residue matrix is an  $(m \times m)$  Hermitian matrix given by:

$$A_k = Q_k C \left[ \sum_{s=1}^n \frac{Q_k^{-T}}{-\lambda_k - \lambda_s} + \frac{Q_k^T}{-\lambda_k - \lambda_s} \right] \quad (10)$$

The contribution to the residue from the  $k$ th mode is given by:

$$A_k = \frac{Q_k C \bar{Q}_k}{2\alpha_k} \quad (11)$$

Where  $\alpha_k$  is the negative of the real part of the pole. When the damping is light, the remaining term is proportional to the mode shape and can be expressed as:

$$Q_k = \phi_k \gamma_k \quad (12)$$

Therefore,

$$A_k \propto Q_k C \bar{Q}_k = \phi_k \gamma_k C \gamma_k^T \phi_k^T = d_k \phi_k \phi_k^T \quad (13)$$

Where  $d_k$  is a scalar constant, and  $\phi_k$  and  $\gamma_k$  are the mode shape vector and the modal participation vector, respectively. Thus, in the case of a lightly damped structure the response spectral density can always be written as:

$$G_{yy}(j\omega) = \sum_{k=1}^n \frac{d_k \phi_k \phi_k^T}{j\omega - \lambda_k} + \frac{\bar{d}_k \bar{\phi}_k \bar{\phi}_k^T}{j\omega - \bar{\lambda}_k} \quad (14)$$

This is a modal decomposition of the spectral matrix.

Then, the PSD matrix of output signal is decomposed by taking the SVD of the matrix.

$$\hat{G}_{yy}(j\omega_i) = U_i S_i U_i^H \quad (15)$$

Where the matrix  $U_i = [u_{i1}, u_{i2}, \dots, u_{im}]$  is a unitary matrix holding the singular vectors  $u_{ij}$ , and  $S_i$  is a diagonal matrix holding the scalar singular values  $S_{ij}$ .

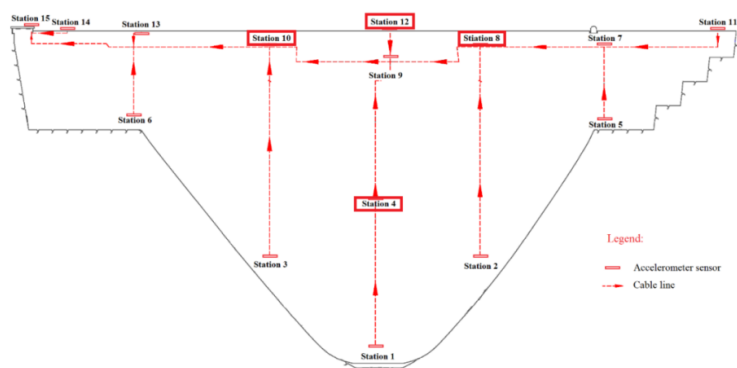
Frequencies of each pick in the first singular value are equal to frequencies of the record. In this method, to choose the correct picks, the Modal Assurance Criterion (MAC) was used [7]. The MAC compares the relationship between two complex mode shape vectors  $\phi$  and  $\psi$  by linear comparing:

$$MAC = \frac{|\psi^T \phi|^2}{(\psi^T \psi)(\phi^T \phi)} \quad (16)$$

Continues Wavelet Transform (CWT) was used separately to decompose response signal to all identified frequencies. Eliminating of modes interference and considerably increasing the accuracy of calculated damping ratios are the advantages of this process. Then, half power method applied to PSD of each single frequency signal to calculate damping ratios

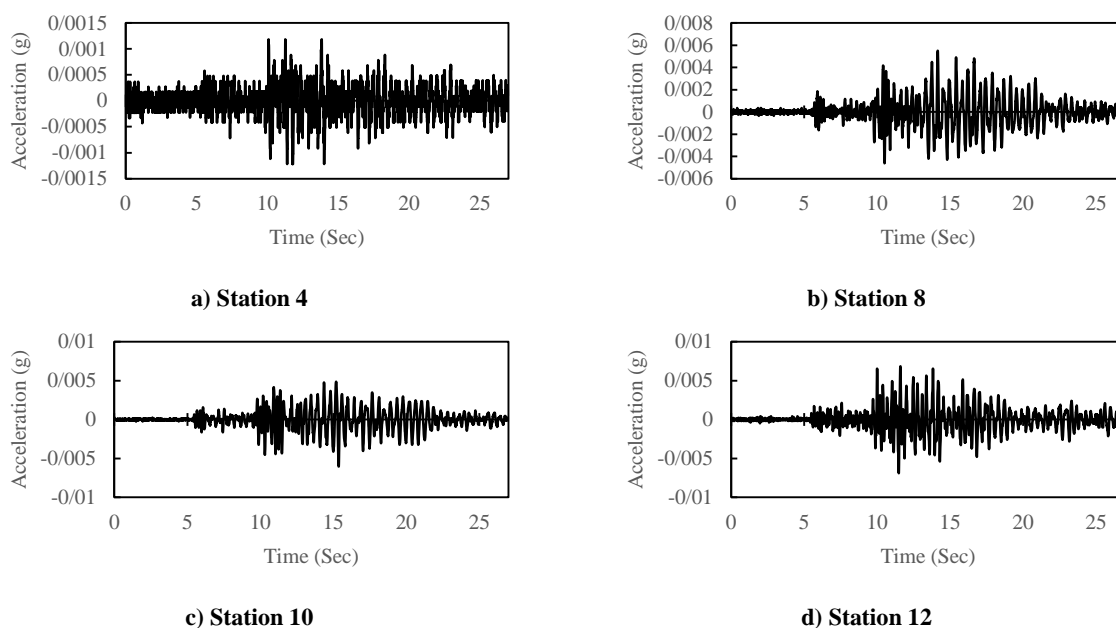
### 3. IDENTIFICATION OF MODAL PARAMETERS OF KARUN III DAM

Karun III is one of the biggest double curved arch dam in Iran. The height and the crest length are 205 m and 462 m respectively, and its thickness varies from 29 m at the base of the crown cantilever to 5.5 m at its crest level. An array of 15 accelerometers were installed on Karun III dam body to record dam responses to ground motions. As is represented in Figure 2, four accelerometers were selected to study identification of modal parameters of the dam.



**Figure 1- Location of accelerometers on Karun IV dam body**

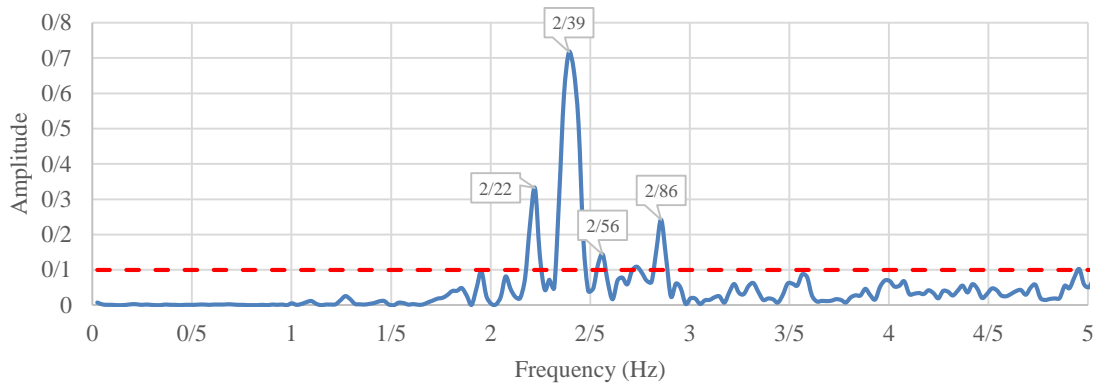
The recorded acceleration at 04 August 2012 was selected to investigate modal parameters. Two other recorded events also were selected to verify the results obtained by processing the records of the first motion. The recorded accelerations in stations 4,8,10 and 12 are shown in Figure 3.



**Figure 2- Recorded accelerations. A) station 4. B) station 8. C) station 10. D) station 12.**

### 3.1. FREQUENCY IDENTIFICATION OF KARUN III DAM WITH PP METHOD

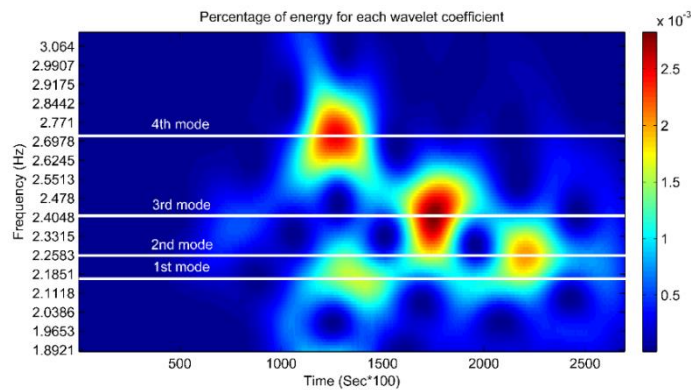
The peak picking (PP) method is the simplest method in system identification methods. It is based on power spectral density matrix of the accelerations. Here, there are four recorded motions from four different stations which have their own PSD spectrum. In the first step all four PSD spectrum should be normalized and then average spectrum should be calculated. The averaged normalized power spectral density (ANPSD) spectrum can be used to calculate frequencies of the structure as shown if figure 4.



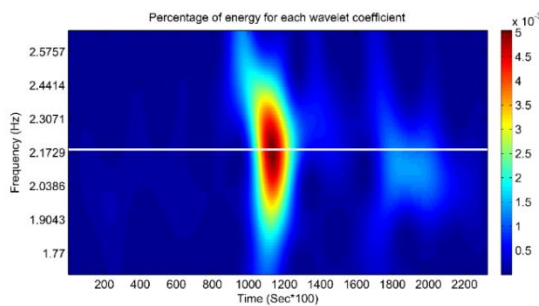
**Figure 3- ANPSD of the recorded motion**

**3.2. FREQUENCY IDENTIFICATION OF KARUN III DAM WITH CWT METHOD**

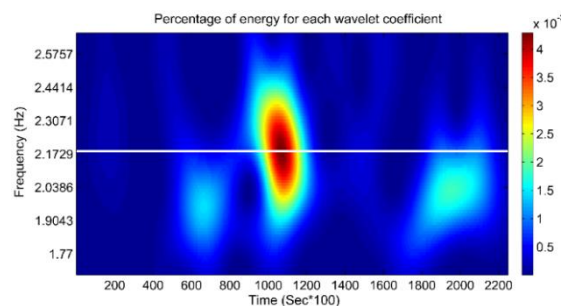
The calculated CWT for the record 1 is shown in Figure 5. As shown in Figure 5, there are four peaks of energy and each of them could be assumed system frequency, but the energy of the first identified mode is not as much as the others. Therefore, to find the first mode of the system accurately, two other records were used. Figure 6 and 7 represent the first mode which were identified with record 2 and 3 respectively. As it can be seen, these figures verify the selected modes of record 1.



**Figure 4- CWT of first motions and identified frequencies**



**Figure 6- CWT of third motion and identifying first mode**

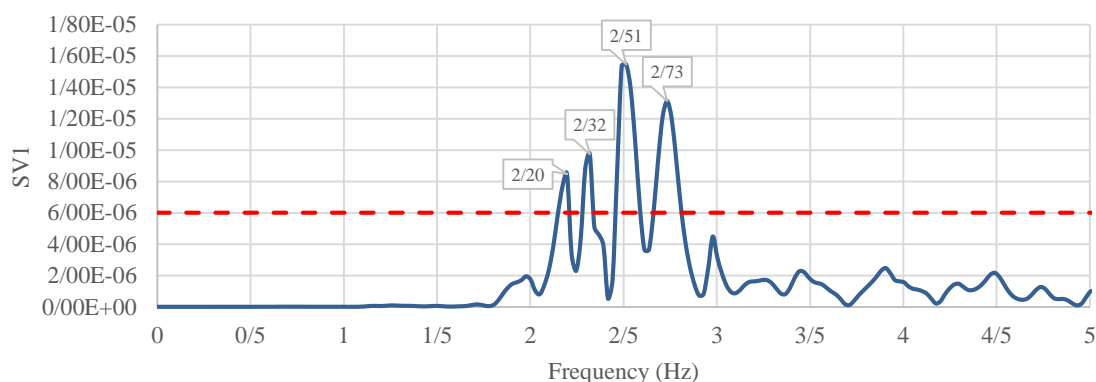


**Figure 5- CWT of second motion and identifying first mode**

The identified modes of the Karun III dam by CWT method due to ambient vibrations are briefly shown in Table 1.

### 3.3. FREQUENCY IDENTIFICATION OF KARUN III DAM BY FDD METHOD

Frequencies of the system with PP and CWT methods were extracted. Here, in this section, FDD method was applied to calculate frequencies of the Karun III dam. As explained, FDD method decomposes acceleration record to its singular values and the peaks which appear in first singular value represent frequencies. Figure 8 shows first singular value of recorded motion and four identified frequencies.



**Figure 7- first singular value of recorded motion**

Three different methods (PP, CWT and FDD) in frequency domain and time-frequency domain were used to calculate frequencies of the Karun III dam from recorded motions. The results of all three methods briefly are presented in table 1 below.

**Table 1- identified modes of Karun III dam**

Modes	Frequency (Hz)			Average (Hz)	Error(Percent)
	PP	WT	FDD		
1 <sup>st</sup>	2.22	2.18	2.20	2.20	1.80
2 <sup>nd</sup>	2.39	2.26	2.32	2.32	5.44
3 <sup>rd</sup>	2.56	2.42	2.51	2.50	5.47
4 <sup>th</sup>	2.86	2.72	2.73	2.77	4.89

### 3.4. CALCULATING DAMPING RATIOS

Half power method was applied to calculate damping ratios in four identified modes. Because the identified modes, especially the first two modes are close, modes interference causes significant errors in calculating damping ratios. Therefore, the dam responses were decomposed to identify frequencies. These frequencies were used to eliminate modes interference in order to use half power method to calculate damping ratios exactly. Damping ratios calculated are represented in Table 2.

**Table 2- calculated damping ratios of Karun III dam**

Modes	Damping ratio (%)
1 <sup>st</sup>	1.66
2 <sup>nd</sup>	1.58
3 <sup>rd</sup>	1.02
4 <sup>th</sup>	1.12

## 4. CONCLUSIONS

In this paper PP, CWT and FDD methods were used to calculate modal parameters of Karun III dam. Because the recorded responses on dam body are ambient vibrations and cannot excite all modes of the dam clearly, more than one motion was used to find weak modes.

The first identified frequency by three methods has 1.8% difference and its average magnitude is 2.2. the first mode is the most important one in system identification process and 1.8% error demonstrates that the all three methods have almost similar results. the second identified frequency and its tolerance are 2.32 Hz and 5.44% respectively. In all 4 identified modes, error tolerance percentage for different methods is lower than 5.47% which shows excellent accuracy for identification process.

The signal decomposition technique which was used to eliminate modes interference showed good results. In addition, damping ratios were calculated separately for each mode. This method significantly increases the accuracy of calculated damping ratios.

## 5. ACKNOWLEDGMENT

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