Smoothed particle hydrodynamics for morphology changes and a non-newtonian fluid

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Abstract

The interaction between bed-load sediment and water flow is an important topic of great concern which should be modeled for a wide range of hydrodynamic systems. In this paper, smoothed particle hydrodynamics (SPH) is utilized for the simulation of mudflow with non-Newtonian behavior and dam-break propagation over an erodible bed that sediment particles as weakly compressible flow is treated as a non-Newtonian fluid using Bingham-Cross model coupled with the Newtonian treat (Owen’s relation) at the interface. To cope with the difficulties arisen from different densities, the continuity and momentum equations are rather modified so that the interactions between the sediment and water are accurately modeled. To validate the Bingham-Cross model, a mudflow test case is studied and compared with other experimental and numerical studies. Then, the two-phase model is used to simulate the dam-break models with PVC bed materials. Comparisons are then made with the available experimental data indicating that the defined SPH model provides the sensible prediction for a test case.

Keywords: Smoothed Particle Hydrodynamics, Dam-break problem, Non-Newtonian fluid, Mudflow, Movable bed.

1. INTRODUCTION

The study on changes in the shape of channels and river beds is necessary for understanding bed load transports. It is clear that the knowledge of random particle movement of bed material transport is also essential for understanding the river morphology, which depends on the pattern of sediment transfer along the river with the local erosion and deposition. Sediment transportation in hydrodynamics is of great industrial and environmental importance which is complicated to model due to its complex boundary conditions and random particle movements. The crucial characteristics of loose boundary problems is the interaction between the fluid and sediment, which is the erosion and sediment transport problems. This cannot be treated in an isolation from the hydrodynamics. Sediments form a passive medium that only reacts to the applied forces [1].

Dam-break with a moveable bed is a challenging problem needed to be simulated in particular when a large amount of sediments is propagated mainly due to dam failure or other defense structures. This may result in a large-scale modification of the valleys morphology along with the environmental and geological changes which significantly increases the hazardous damages to the mankind and urban infrastructures [2]. Modeling of the dam-break problem over mobile bed, based on the Eulerian methods, is rather complicated. The predicted models should be able to accurately evaluate the bed movement and the free-surface variation. Smoothed particle hydrodynamics (SPH) is a fully Lagrangian and meshless method. In this method, each particle carries an individual mass, position, velocity and other physical quantities. The Lagrangian nature of SPH is suited for simulating problems with large deformation, e.g., dam-break, as there is no special treatment needed for free surface.

In this paper, the open-source SPHysics code [3] will be modified into two phases, where the sediment phase is considered as non-Newtonian. Moreover, the viscosity of sediment in the momentum equation is simulated using Bingham fluid with Cross model. The water–sediment interface is modeled using Owen’s equation, leading to precise study of the water–sediment interface. Therefore, a case of mudflow is studied to validate the SPH code on Bingham fluid behavior and is compared with the experimental study of Komatina and Jovanovic [4] and numerical studies of Shao and Lo [5] and Capone [6]. Then, the case of dam-break problem is studied and compared with the experimental results of Spinewine [2] and two numerical results of Shakibaeinia and Jin [7] and also Razavitoosi *et al.* [8].

2. SPH METHOD

The SPH method generally applies an integral interpolation of a function f, defined over a domain of interest Ω, allowing f to be estimated in terms of its values in the surrounding domain. The value of f at location $\boldsymbol{r}$ can be written as a convolution product of the function f [9]:

$$f(\boldsymbol{r}) \approx < f(\boldsymbol{r}) > = \int_{\Omega} f(\boldsymbol{r}')W(\boldsymbol{r}-\boldsymbol{r}',h)d\boldsymbol{r}' \quad (1)$$

The transition to a discrete domain is obtained by approximating the integral of equation 1 by a summation. The value of quantity f relative to the particle i located at the point $\boldsymbol{r_{ij}} = \boldsymbol{r_i} - \boldsymbol{r_j}$ can be written as:

$$f(r_{ij}) = \sum_j \frac{m_j}{\rho_j} f_j W(r_i - r_j, h) = \sum_j \frac{m_j}{\rho_j} f_j W_{ij} \quad (2)$$

where $W(\boldsymbol{r}-\boldsymbol{r}',h)=\boldsymbol{W_{ij}}$ is called the smoothing kernel and h is the smoothing length, $d\boldsymbol{r}$ is a differential volume element, m is the particle mass and f_j denotes the value of f at the point occupied by particle j.

The SPH continuity and momentum equations of the Lagrangian form of the Navier–Stokes equation can be obtained using the following formulations [9]:

$$\frac{d\rho_i}{dt} = \sum_j m_j u_{ij} \nabla_i W_{ij} \quad (3)$$

$$\frac{dv_i}{dt} = -\sum_j m_j \left(\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2} \right) . \nabla_i W_{ij} + g + \sum_j m_j \left(\frac{4\upsilon r_{ij} u_{ij}}{(\rho_i + \rho_j)\left(|r_{ij}|^2 + \iota^2\right)} \right) . \nabla_i W_{ij} + \sum_j m_j \left(\frac{\overline{\tau}_i}{\rho_i^2} + \frac{\overline{\tau}_j}{\rho_j^2} \right) . \nabla_i W_{ij} \quad (4)$$

where m_i is the mass of particle i, P_i is the pressure that particle experienced, u is the velocity vector of particle, ρ is the density, $\boldsymbol{g}$ is the gravity acceleration, v is the kinematic viscosity, ι is a very small number avoiding the term become infinity and $\bar{\tau}$ represents the SPS stress tensor.

Due to the discontinuity in density in multi-fluid simulation, Greiner et al. [10] proposed a new method for multiphase problems. The algorithm requires sweeps over the particles to determine the volume distribution, density, rate of change of volume (continuity equation), and the acceleration. The following simple algorithm (according to Monaghan and Rafiee [11]) is capable of handling the density ratios that normally occur. The continuity and momentum equations in this algorithm are:

$$\frac{d\rho_i}{dt} = -\rho_i \sum_j \frac{m_j}{\rho_j} (u_j - u_i) . \nabla_i W_{ij} \quad (5)$$

$$\frac{dv_i}{dt} = -\sum_j m_j \left(\frac{P_i + P_j}{\rho_i \rho_j} + R_{ij} \right) . \nabla_i W_{ij} \quad (6)$$

The following form of R_{ij} is used according to Monaghan [12] and Grenier et al. [10]:

$$R_{ij} = K * \left(\frac{\rho_d - \rho_l}{\rho_d + \rho_l} \right) \left| \frac{P_i + P_j}{\rho_i \rho_j} \right| \quad (7)$$

where ρ_d and ρ_l are the reference density of the denser and the lighter fluid, respectively, and K is a free coefficient.

In multiphase systems viscosity, discontinuity happens when phases have different viscosities. Therefore, we use Owen's equation (see [13]) for an interface viscosity which is used in the laminar viscosity term of equation 4 instead of viscosity parameter v.

$$\vartheta_{mix} = \frac{\vartheta_{fluid}}{1 + C\frac{\rho_s}{\rho_f}} \quad (8)$$

where ρ_s and ρ_f are the sediment and fluid density, respectively, and C is the concentration of solid particle which is defined as:

$$C = \frac{\sum_{j \neq i} \delta_{sf} W_{ij}}{\sum_{j \neq i} \delta_{sf} W_{ij} + \sum_{j \neq i} (1 - \delta_{sf}) W_{ij}} \qquad \delta_{sf} = \begin{cases} 0 & \text{for fluid particle} \\ 1 & \text{for solid particle} \end{cases} \quad (9)$$

For modeling the sediment herein, the Bingham fluid assumption is used which is due to the non-Newtonian behavior of shear stress distribution for the sediment particles. Bingham model can be stated on two different behaviors. The first is the solid behavior which is below the yield stress point and second is above this point as fluid behaves similar to the Newtonian fluid with a constant viscosity.

In this study, the numerical computation effective viscosity μ_{eff} is used to simulate the Bingham fluid behavior as:

$$\mu_{eff} = \mu_B + \frac{\tau_y}{D} \tag{10}$$

To define the effective viscosity, the general Cross model is as follows according to [5]:

$$\frac{\mu_0 - \mu_{eff}}{\mu_{eff} - \mu_\infty} = (KD)^m \tag{11}$$

where μ_B and τ_y are the Bingham viscosity and yield stress, respectively. Moreover, μ_0 and μ_∞ are viscosity at very low and very high shear rates, respectively; K and m are constant parameters. The shear rate which is simplified in 2D is defined as:

$$D = \sqrt{2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2} \tag{12}$$

Considering *m* in equation 11 as unity, the effective viscosity in a Cross model is expressed as:

$$\mu_{eff} = \frac{\mu_0 + K\mu_\infty D}{1 + KD} \tag{13}$$

By comparing the above equation with equation 10, the other parameters under $KD >> 1$ are defined as:

$$K = \frac{\mu_0}{\tau_y} \text{ and } \mu_\infty = \mu_B \tag{14}$$

The remaining unknown parameter in the equation 13 is the viscosity at low shear rate (μ_0). This viscosity should have a large value to fix and freeze the particles when the shear rate is very low. So, in this study, the viscosity at low shear rate is set as $\mu_0 = 10^3 \mu_\infty$. Due to continues variation of effective viscosity and in order to avoid the numerical instability, the Cross model (equation 13) is suggested [5].

The sediment–water mixture is the two-phase flow that sediment is treated as a non-Newtonian and water is a Newtonian fluid. In this paper, we use the effective viscosity, which is identified as Cross model, for each sediment particle, and for the interface between two phases we use equation 8.

To calculate the pressure term and to avoid solving the Poisson equation of incompressible fluids at each time step, in weakly compressible form of SPH, the Tait's equation of state is used [14]:

$$P = B\left[\left(\frac{\rho}{\rho_0}\right)^\gamma - 1\right] + \chi \tag{15}$$

Where ρ_0 is the reference density and $B = c_s^2 \rho_0 / \gamma$ where $\gamma = 7$ is typically used to induce strong pressure response to density variations, such that the density variation remains small enough for the fluid volume to be conserved and c_s being the corresponding speed of sound.

In two-phase flows, [15] suggested additional term back pressure, *X*, in addition to modify the bottom phase pressure. This back-pressure term is the column of water above each SPH sediment particle which is updated at each time step. This term is added for two-phase flow to ensure that the excessive pressure of water corresponding to the column of water above the sediments is calculated for sediment, while it is zero for water [15]. This numerical simulation could be found in [16] in detail.

3. Non-Newtonian Fluid

The Bingham fluid as a non-Newtonian fluid is studied by comparison of the dam-break SPH model using Cross model with the experimental [4] and numerical [5,6] models with the ISPH (Incompressible SPH) and SPH methods. In these studies, channel bed slope $S_0=0.1\%$ is considered. The density of mudflow is 1200 Kg/m^3, the yield stress $\tau_y = 25Pa$ and viscosity $\mu_B = 0.07 Ns/m^2$ are reported by [4].

Figure 1 shows the propagation of the Bingham fluid in present study in comparison with the numerical study by [5] that used ISPH method at t=0.1, 0.3, and 0.6s.

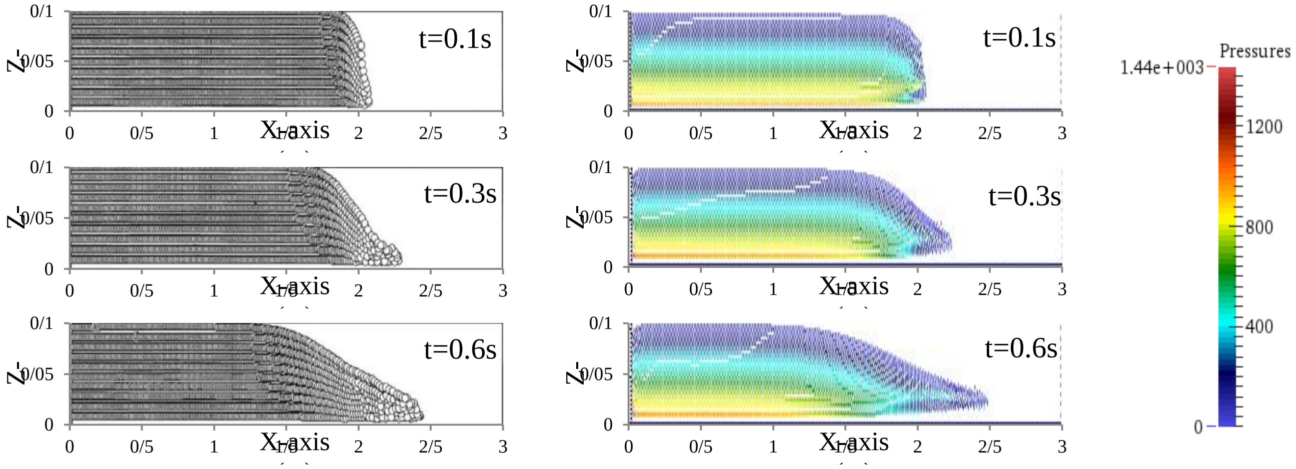

Figure 1. Mudflow profile in dam-break problem; left column is studied by [5]; right column is the present study

The dimensionless comparison on the experimental [4], Capone's SPH numerical [6] studies and the present model illustrate in figure 2. This figure shows the present model has almost proper behavior in compare with experimental study. In this figure, H is the initial depth of fluid and x is the propagation length of fluid. T and ε are the dimensionless form of time and propagation length, respectively.

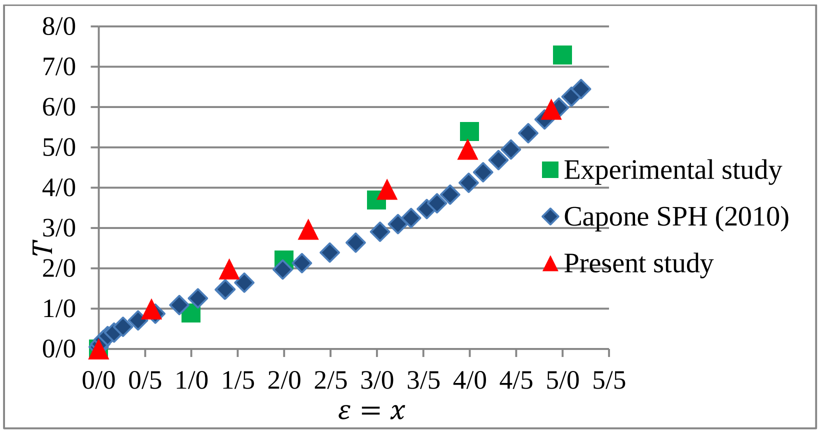

Figure 2. Propagation of mudflow in compare with other studies

4. DAM-BREAK MODEL ON THE PVC BED

The experimental model of Spinewine [2] is exerted to simulate the dam-break problem. In this experiment, a dam-break model is studied, in which the PVC materials with diameter of 3.9 mm are used for the saturated sediment of the bed. The study is considered in the flume with the length of 6m and a gate with a negligible friction factor, located at the middle of flume and separates upstream and downstream. In these models, water flows to downstream due to the gate removal, eroding the bed and subsequently changing the topography. The schematic of the problems is presented in Figure 3 and the sediment properties are summarized in Table 1. In these case studies, the particle spaces are 0.01 m and the CPU cost on a workstation, Core i7, 2.20 GHz, RAM 8.0 GB for 39,748 particles is about 5 h.

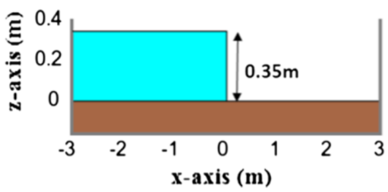

Figure 3. Scheme of dam-break model

Table 1. Properties of sediments for the bed material

Material	Mean diameter of sediment (m)	Specific density (kg/m^3)	Bulk density (kg/m^3)	Friction angle (°)
PVC	0.0039	1580	1336	38

In this test case, the flume bed is cover with the PVC pellets sediment which has a flat surface bed. Figure 4 presents the SPH dam-break of this test case at 0.25, 0.50, 0.75 and 1.0s compared with experimental studies.

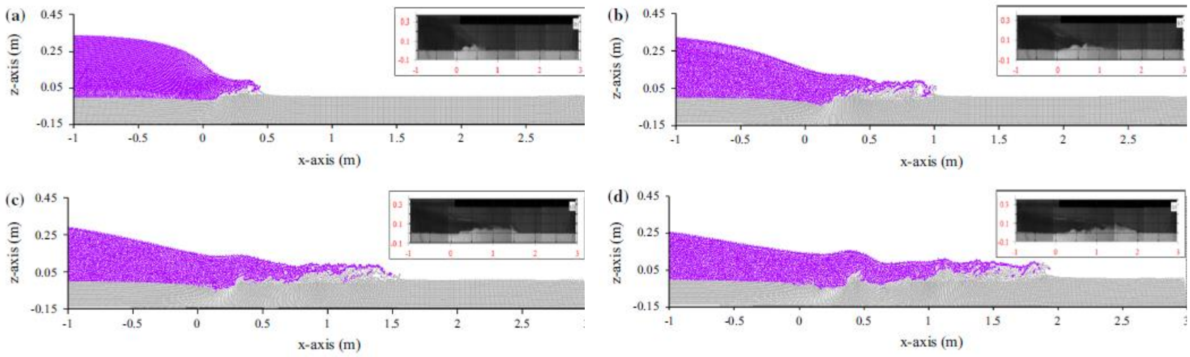

Figure 4. The snapshots of dam-break problem on PVC bed study at t=0.25, 0.50, 0.75, 1.0s (compared with [2])

As the figures show, after removing the gate, water flows to the downstream while the shear stress increases and it becomes unstable with erosion. Large erosion is happened near the gate while the sediments distribute to the downstream and deposit far from the gate. Increasing the bed elevation is occurred behind the wave front due to the erosion beneath the front. As the figure shows, the bed is not decreased monotonously but series of humps and troughs are generated due to the wave propagation.

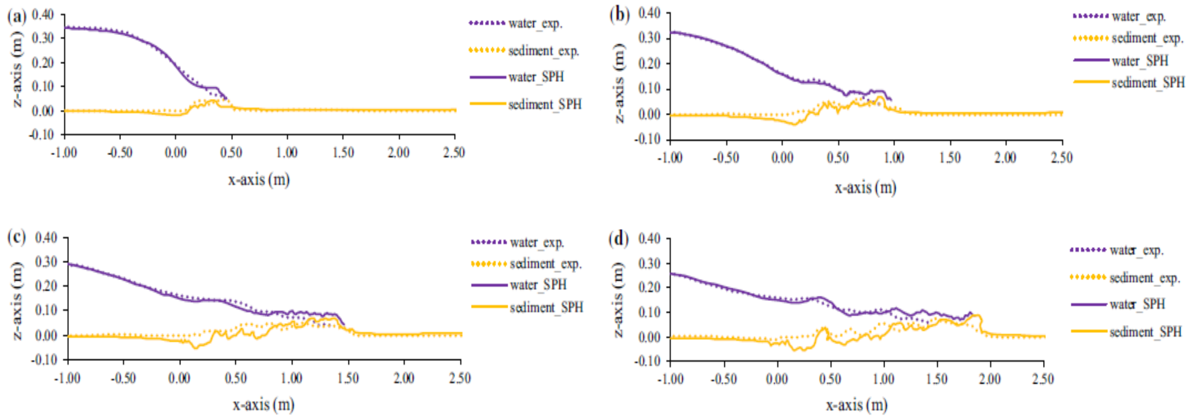

Figure 5. Free surface and bed surface changes on PVC bed study at t=0.25, 0.50, 0.75, 1.0s (compared with [2])

Here, the SPH simulation is in a good agreement with experimental studies as also shown in Figure 5. This Figure compares the SPH surface elevation with the results of [2]. As shown, the rheological model of bed is reasonably treated, and the results are in good agreement. It is worth mentioning that the snapshots of experimental model are taken from the side of flume. This may be the reason of discrepancy between SPH and experimental snapshots.

To quantify the existing error in the calculation of water and sediment surface elevation, the relative error norm (ε_{L2}) is defined to provide a good measureable precision. RSM error is defined as [7]:

$$\varepsilon_{L2} = \left(\frac{\sum_{i=1}^{N}(\Delta H)_i^2}{\sum_{i=1}^{N}(H)_i^2}\right)^{1/2} \tag{16}$$

where ΔH is the deviation of numerical water surface/sediment bed surface elevation from the experimental values (H) and N is the number of points at which the elevations are compared. The sediment and water surfaces of numerical results are compared with their related points in a deformed area according to Shakibaeinia and Jin [7]. The errors of the free surface and bed surface SPH model according to the experimental ones are illustrated in Table 2. The results indicate the reasonable behavior of the rheological model is used in this article.

Table 2. The relative error, ε_{L2}, between experimental and numerical results

Time(s)	0.25	0.50	0.75
Error in free surface	0.046	0.034	0.045
Error in bed surface	0.057	0.096	0.105

Figure 6 presents the difference between three models of simple Newtonian Owen's relation (Figure 6b), Bingham–Cross model (Figure 6c), and Bingham–Cross model coupled with Owen's relation (Figure 6d).

As these figures show, the Bingham–Cross model coupled with Owen's relation has a better result at t=0.25s and correctly treated as a PVC sediment bed with time proceeding in comparison with experiments. In this model, equation 8 is used for the interface of water and sediment (Figure 6d). In this method, the sediment particles have granular behavior where sediment particles are pushed and eroded with water particles. Therefore, there are some isolated particles for fluid in sediment area and for sand in water area. Although, the behavior of sediment using equation 8 results in a noisy pressure distribution at the interface (See Figure 7) but according to Figure 6 it gives a better interface in comparison with experiment. Figures 8 and 9 present the relative errors of free surface and bed surface, respectively.

Figure 8 presents a better result of current study than the model proposed by [7]. As this figure shows, the error of both models is not significant at t=0.25s but with time proceeding this error increases until t=0.5s and after that the error becomes constant.

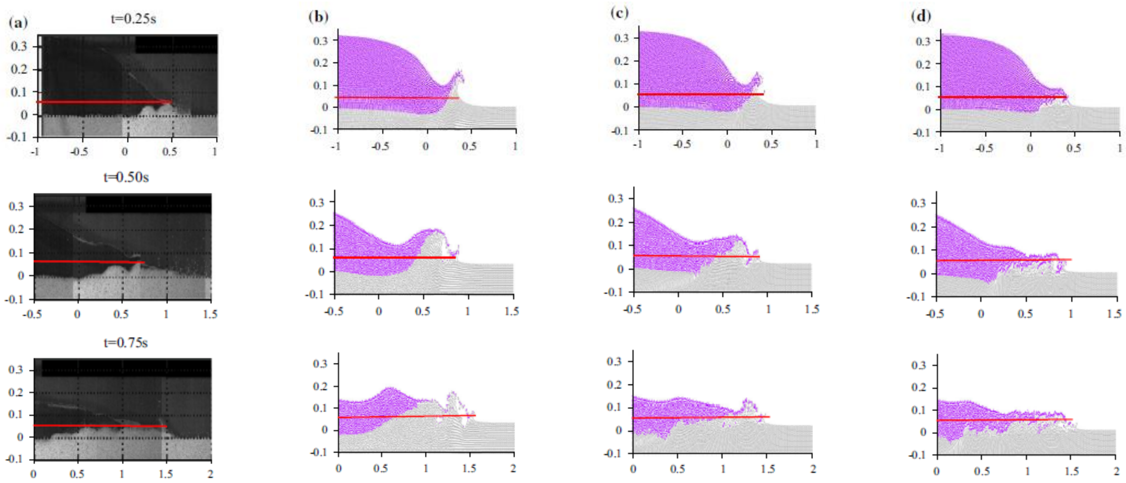

Figure 6. Dam-break problem over PVC bed; a) experiment of [2]; b) the Newtonian Owen's relation; c) the Bingham-Cross model; d) the Bingham-Cross model coupled with Owen's relation

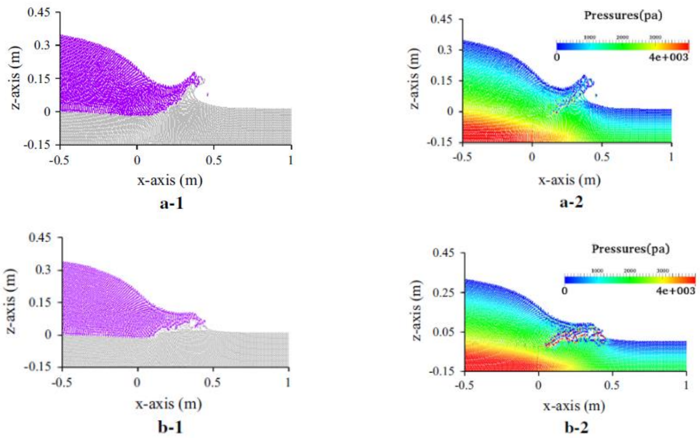

Figure 7. Particle distribution and pressure field for a) Bingham-Cross model and b) Bingham-Cross model coupled with Owen's relation at the interface

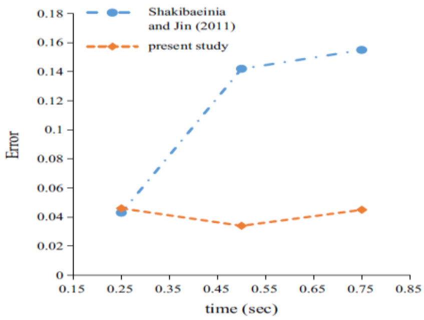

Figure 8. Comparison between the relative errors for the free surface

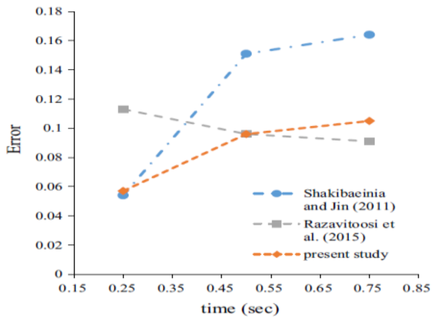

Figure 9. Comparison between the relative errors for the sediment bed surface

As Figure 9 shows, at t=0.25s, the error of sediment bed surface of current study is close to MPS and less than the [8] model who used only Cross model. With the time proceeding, the error of the current study increases less than the MPS model and close to the errors of [8] SPH model to t=0.75s. The errors of both SPH models (present and [8] studies) with the time proceeding are close to each other and less than the model of [7]. The results for t=1.0s have not reported by these researchers.

5. CONCLUSIONS

Here, we used the SPH method to model violent flow over a movable bed where sediment is considered as a Bingham fluid. The study of mudflow was done and compared with the experimental and numerical studies to validate the Bingham-Cross model. The comparison was illustrated that the Bingham-Cross model has proper ability to simulate the mudflow. In the two-phase model, the Bingham–Cross model that coupled with Owen's relation is used to simulate the sediment bed with a careful study of the water–sediment interface. This study shows, in water-sediment models, sediment at the interface does not treat as the Bingham behavior exactly. We have used available experimental and numerical methods to validate our results.

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