Global Sensitivity Analysis of Concrete Gravity Dams Subjected to Seismic Excitation

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Abstract
In this study, global sensitivity analysis is performed to determine relative importance of random variables and their interaction effects onto the response of concrete gravity dam model. The dam-foundation reservoir system is considered in this research to investigate the effect of input variables in the mentioned system. Material properties of concrete and foundation are assumed as random variables to tackle epistemic randomness involved in these models. The main novelty of this paper is to quantify the effect of each uncertain input variable and its interactions with other variables on the response of gravity dams. Most of the studies in the field of dam engineering focused on parametric or local sensitivity approaches. Nonetheless, these methods have the following drawbacks. First, they are based on only one point in the variable space. Second, the interaction between uncertain variables cannot be determined in parametric and local sensitivity analyses. On the other hand, in global sensitivity analysis, the interactions of random variable with other random variable are determined over the whole input space. In this research, variance-based sensitivity analysis is utilized as the model free global sensitivity approach to accurately quantify the effect of each random variable in the response of concrete gravity dams.
Keywords: Concrete Gravity Dam, Seismic Excitation, Global Sensitivity Analysis, Limit-State, Sensitivity Measure.

1. INTRODUCTION

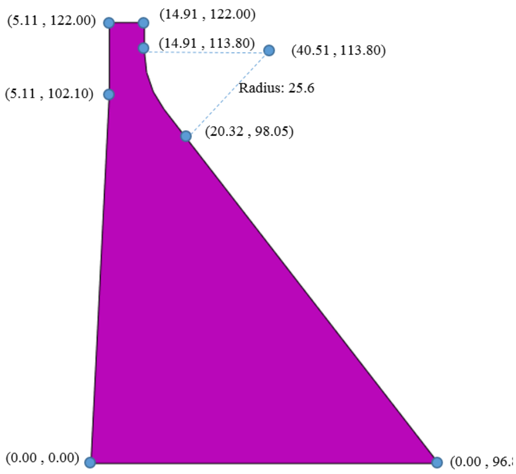
The safety level of infrastructures is a critical concern and traditional methods consider simply safety factors for design and safety evaluation. As consequences of infrastructure including dams can be catastrophic, it is important to accurately evaluate the safety level of these systems. Furthermore, the failure of these infrastructures leads to substantial human casualties and economic losses. Consequently, employing more reliable approaches along with improved determination of loading conditions will result in more realistic assessments of safety. The complexity of Dam's behavior and a plethora of uncertainty involved in their numerical models has to be tackled specifically in seismic analysis. The probabilistic approach is an effective method to take into account the aforementioned difficulties. Uncertainties can be due to material properties, geometry of structure, environmental phenomenon, loads, and other factors. The sources of uncertainty can generally be categorized into aleatory and epistemic randomness. An aleatory uncertainty is irreducible randomness due to the nature of phenomenon, while an epistemic uncertainty is related to a lack of knowledge and is measurable and reducible.

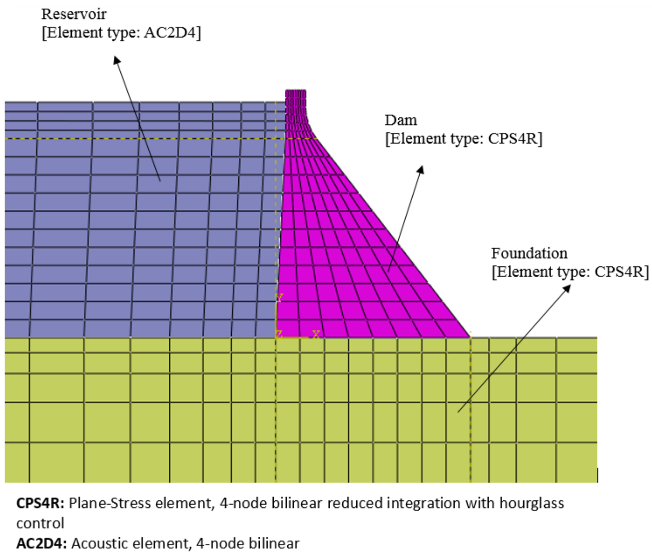
Many previous studies have been determined an accurate assessment of dam safety under different failure modes, loading conditions, and various random variables were considered [1-4]. Nonetheless, many problematic issues have remained unsolved in these studies. They were unable to sufficiently determine the quantitative importance of the variables and utilized insufficient reliability methods and inappropriate performance criteria. Some studies used parametric analysis as a sensitivity analysis by applying perturbation to the mean of the random variables [5]. The local sensitivity analysis is based on the derivative of each random variable in a specific point, which can be the mean or the design point depending on the methods employed [6, 7]. In global sensitivity analysis, the influence of entire range of variables is neglected. In addition, crude sampling methods, such as MCS, require a large number of samples to obtain acceptable accuracy. Hence, employing more efficient sampling methods is inevitable and it is intended to implement a reliability method on models with an intrinsic high computational cost [8-9].

In this investigation, the relative importance of each random variable along with its interaction with other random variables is determined. The variance-based sensitivity analysis (VBSA) is used for quantifying the importance of random variables. As the computational cost of each deterministic analysis is high, it is inevitable to use efficient sampling as the basis of the VBSA. For this purpose, Latin Hypercube sampling which is an efficient less time consuming sampling method than MCS is used.

2. MODEL DESCRIPTION

For this research, the tallest monolith of the Pine Flat Dam, a concrete gravity dam in Central California, is selected as a case study. The geometry of dam's body and the elevation of the reservoir are presented in Figure 1(a). The concrete properties are assumed to be linearly elastic and isotropic. The Pine Flat dam is modeled using two-dimensional finite element method in a plane-strain analysis. The dam-foundation-reservoir interaction is considered in the model. The length of reservoir is assumed five times the dam height and non-reflective planar boundary condition is assigned as far-end boundary condition. The foundation rock is presumed to be massless. The finite element model of dam-foundation-reservoir is shown in Figure 1 (b). The material properties of concrete, water and rock foundation is presented in Table 1.





(a) (b)

Figure 1. Geometry, finite element model and element type of the case study

Table 1- Material properties of dam-foundation-reservoir system

	Material	Value
Concrete	Elasticity modulus	30 (GPa)
	Poisson's ratio	0.2
	Density	2400 (kg/m^3)
	First and third mode damping ratio	5 %
Water	Density	1000 (kg/m^3)
	Bulk modulus	2.07 (GPa)
Foundation Rock	Elasticity modulus	30 (GPa)
	Poisson's ratio	0.33

For the seismic analysis, Taft ground motion, Kern County, 7/21/1952, Taft Lincoln School, 111, is utilized in three different level of intensity based on operating basis earthquake (OBE), maximum design earthquake (MDE) and maximum credible earthquake (MCE) having PGA of 0.18, 0.27 and 0.45 g, respectively. Based on damage index defined by Alembagheri and Ghaemian (2013), seismic performance of gravity dam can be pertinent to crest displacement. They utilized nonlinear static pushover and incremental dynamic analysis to quantify the crack state of Pine Flat gravity dam into crest displacement [10]. The crack initiation and ultimate state are considered for damage index determination. In their model, the concrete behavior in uniaxial tension is controlled by tension stiffening and tensile damage (d_t). The degradation of the material stiffness due to damage propagation in terms of cracking normal displacement is considered. Crack initiation state is defined by the first element of dam body, commonly the hill element, exceeding tensile damage. Besides, the ultimate state is indicated by the crack propagation of the dam's neck as well as the other cracked part of the dam body. In these states, the crest displacement has been considered as a performance criterion.

In this study, the performance criteria are defined using aforementioned damage index and tensile over-stressing indices as follows:

$$f_1(\mathbf{x}) = 1 - DI_c < 0 \qquad (1)$$

$$DI_c = \frac{U_{\max}}{U_d} \qquad (2)$$

$$f_2(\mathbf{x}) = S_{all} - S < 0 \qquad (3)$$

Where $f_1(\mathbf{x})$ and $f_2(\mathbf{x})$ are the limit-state functions based on maximum crest displacement (MCD) and tensile over-stressing, respectively. U_{max} and S are the maximum crest relative displacement and tensile stress, respectively. U_d and S_{all} are the thresholds corresponding to displacement and stress limit-states, respectively. The domain $f(\mathbf{x})<0$ represents the failure domain of the responses. These performance criteria controlling the maximum crest displacement and the tensile over-stressing are used to define limit-state functions that are required for the uncertainty analysis.

Material properties of dam and foundation rock are taken as random variables to quantify uncertainty in the current case study. According to engineering judgment and pertinent studies, these random variables are assumed to be uncorrelated and assigned probabilistic characteristics are shown in Table 2. Allocating nearly high value of standard deviation to the random variables is justified by large uncertainties involved in these variables due to lack of experimental and site data. In addition, influence of seismic randomness is assumed using parametric analysis to model uncertainty.

Table 2- Probabilistic characteristics of defined random variables

Random Variable	Mean	Standard Deviation	Probability Distribution
Concrete density (kg/m^3)	2400	480	Lognormal
Elasticity modulus of concrete (GPa)	30	0.6	Lognormal
Ratio of elasticity modulus of rock foundation to concrete (GPa)	0.625	0.216	Uniform
Concrete Poisson's ratio	0.2	0.04	Uniform

3. METHODOLOGY

Variance-based sensitivity analysis (VBSA) determines the influence of each random variable and its interaction on the total variance of response, called first order (S_i) and total sensitivity (S_{Ti}) measures, respectively [11-15]. This method is appropriate for utilizing with complex nonlinear models. The main drawback of this method is its dependence on a number of random variables. The total number of analyses required for this approach is (N/2)*M. *N* and *M* represent the number of samples and random variables, respectively. VBSA was implemented using the Sobol method. The variance of output was decomposed to Sobol indices, which imply the first and total effect of each random variable. An analytical function, presented in Eq. 8, with three random variables has been selected to verify the accuracy of the implemented method. All random variables were uniformly distributed with a minimum and maximum value of $-\pi$ and π, respectively.

$$f(X_1, X_2, X_3) = \sin X_1 + a \sin X_2 + bX_3^4 \sin X_1 \qquad (4)$$

In the above equation, *a* and *b* are constant parameters and assumed to be 7 and 0.1, respectively [16]. The main (first order) and total of Sobol' indices were calculated for different numbers of samples, as demonstrated in Figure 2. In spite of slight differences between the exact and calculated indices, the performance of this implemented approach can be observed. For instance, importance measures for N=1024 were calculated and their comparison with exact values are presented in Table 3.

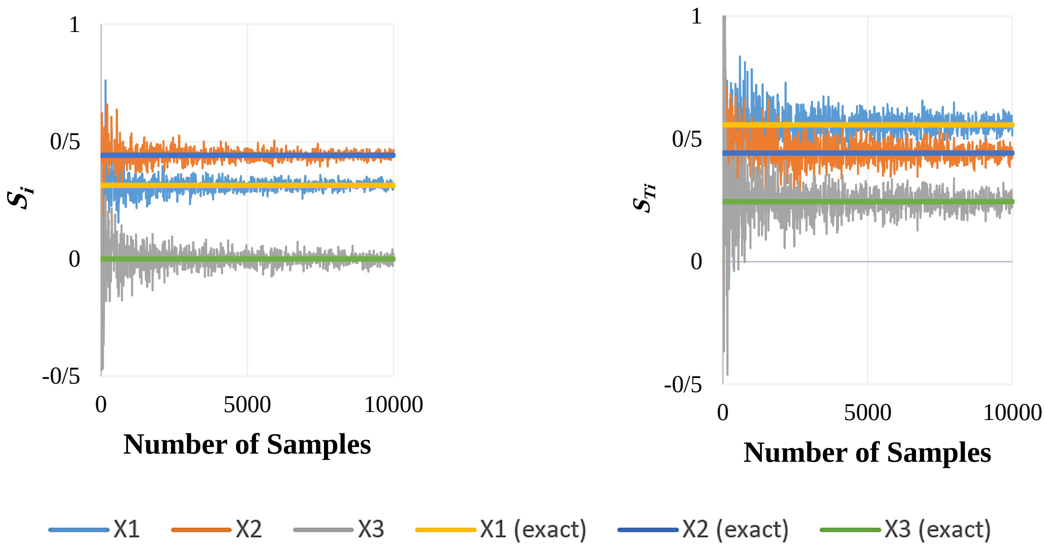


Figure 2. Verification of implemented VBSA (Sobol' indices) tested for equation 4.

Table 3- Comparison of exact and calculated Sobol' indices tested for equation 4.

	Exact value		**Calculation for N=1024**	
	S_i	S_{Ti}	S_i	S_{Ti}
X_1	0.3138	0.5574	0.3140	0.5351
X_2	0.4424	0.4424	0.4468	0.5002
X_3	0	0.2436	0.0064	0.2847

Utilizing Monte Carlo sampling in complex numerical models is not practical since a large number of samples are required to achieve an acceptable confidence level. Consequently, it is recommended to employ more efficient sampling approaches such as importance sampling, adaptive importance sampling, directional sampling, and Latin hypercube sampling (LHS) [17]. LHS was selected for this research since its efficiency and simplicity of implementation. On the other hand, the first step for implementation of importance and adaptive sampling is to find the design point, which is problematic as limit-state functions tend to be nonlinear.

The LHS is a stratified Monte Carlo sampling that results in filling all the areas of the sample space [18]. LHS was implemented by dividing the cumulative density function (CDF) of each variable into N non-overlapping intervals having equal probability as shown in Figure 3. A value for the corresponding variable was randomly selected in each interval. The number of samples was then equal to the number of intervals. The sample matrix was defined for all the variables and for each interval as follows:

$$x_{i,j} = F_j^{-1}(p_{i,j}) = F_j^{-1}(\frac{i-0.5}{N}) \qquad (4)$$

$$i = 1,...,N;\ j = 1,\dots,M$$

where M is the number of random variables and F^{-1} is the inverse CDF of probability p.

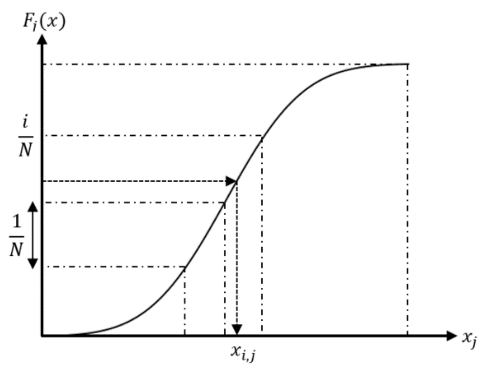


Figure 3. The algorithm of LHS

4. RESULTS

The global sensitivity analysis is performed in the model based on 1,000 samples generated using LHS. The results are obtained separately for two defined limit-states, maximum crest displacement and maximum tensile stress. The first order and total sensitivity measures along with total variance values are calculated based on crest displacement and tensile stress, see Figures 4 to 6.

The result of first order sensitivity measures indicate that for limit-state based on both MCD and maximum tensile stress, the most important variable is Young's modulus of concrete. Furthermore, the second-ranked important variable in the results based on MCD is the ratio of Young's modulus of rock to concrete but in the results based on maximum tensile stress is concrete density. All the results indicate that concrete Poisson's ratio is the least important variables in this model. Despite the dam is asymmetric and also the seismic loading is not identical in U/S and D/S direction, the results of MCD for both of these directions are approximately the same. It implies that the importance measures are independent of these situation and they are only pertinent to the performance functions.

By comparing the first order and total sensitivity measures (in maximum tensile stress limit-state), it can be inferred that the elasticity modulus of concrete has higher impact on the results when its interaction with other variables is considered. For MCD limit-states, it is the ratio of Young's modulus of rock to concrete is significantly affect the response of the dam and is the most important random variables. Based on the total sensitivity measures, the second-ranked important variables are Young's modulus of concrete and the ratio of Young's modulus of rock to concrete for limit-states based on MCD and maximum tensile stress, respectively. The total variance chart implies that the variance involved in the maximum tensile stress results is higher than MCD when considering first order effect. On the contrary, the total variance in total sensitivity calculation indicates that the variance involved in the maximum tensile stress is lower than MCD.

It is noteworthy that the results of model subjected to OBE, MDE and MCE earthquake determined to be identical. The reason is that the model is linear and the by increasing intensity of ground motions, what it is in OBE, MDE and MCE earthquake, only shifts backward and forward the results and the variance of the responses are not changed.

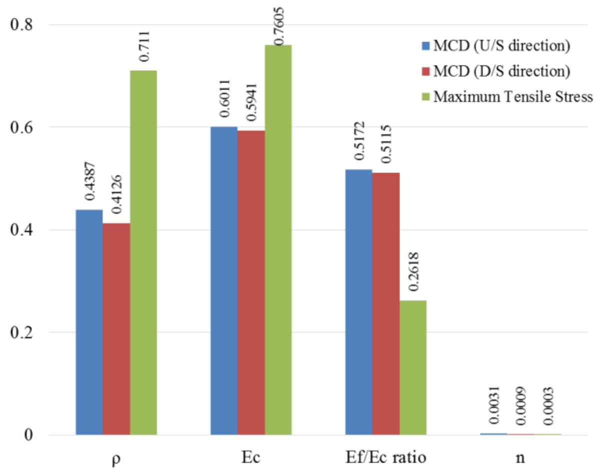


Figure 4. First order sensitivity measure results

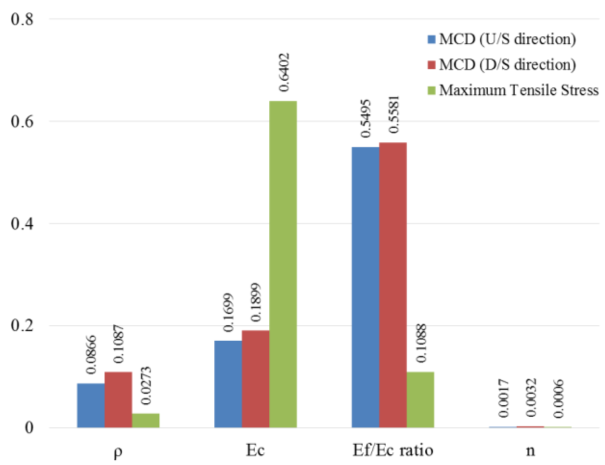


Figure 5. Total sensitivity measure results

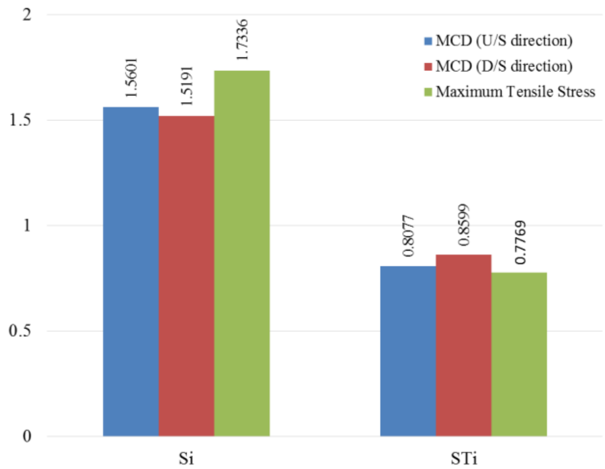


Figure 6. Total variance involved in the results

5. CONCLUSIONS

A probabilistic numerical model of concrete gravity dam subjected to seismic load is studied. In this study, the performance of concrete gravity dam is assumed to be associated with the maximum crest displacement and the maximum principal (tensile stress). For consideration of ground motion effects as a parametric study, the same ground motion with different intensity levels is applied to the model. The objective of this investigation is to accurately quantify the importance of random variables in the dam-foundation-reservoir system. For this purpose, global sensitivity analysis based on Latin Hypercube sampling is implemented. The sensitivity measures are calculated for different limit-state functions. The results indicate that the most important variable based on first order sensitivity measure is concrete Young's modulus. For results of total sensitivity measure, the most important variable is concrete Young's modulus for maximum tensile stress based limit-state and it is the ratio of Young's modulus of rock to concrete for MCD based limit-state.

6. References

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