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Dynamic parameter identification of the Universal Robots UR5

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Abstract— **In this paper, methodology for parameter identification of an industrial serial robot manipulator is shown. The presented methodology relies on the fact that any mechanical system can be written in form linear with respect to some set of parameters. Based on experimental measurements done on the Universal Robots UR5, the presented methodology is applied and the dynamical parameters of the robot are determined in two ways. First by use of the Moore-Penrose pseudoinverse, and then by use of optimization. At the end, the ability of the determined parameters to predict measurements other than the ones used for the identification is shown.**

I. INTRODUCTION

Mathematical model of a real physical system is as good as it can predict what experiments show. In order to have a good model both its structure, meaning taking into account all relevant dynamics, and its parameters must be correct. Some model parameters, like masses and lengths of robot links, can be measured, while others, such as temperature dependent dry and viscous friction, axial and centrifugal moments of inertia or position of center of mass of segments, are almost always unknown and must be identified. However, each parameter can not be separately identified but only linear combinations of them. The vector whose elements are linear combination of parameters that can be identified is called vector of identifiable parameters or vector of base parameters.

In this paper, procedure for determination of base parameters and for their identification is explained. Then, using experimental measurements, the procedure is applied to parameter identification of the Universal Robots UR5 manipulator. At the end, in order to validate the obtained parameters, they are used for predictions of experimental measurements not used for the identification.

II. MATHEMATICAL MODELING

A. Robot dynamics

Differential equations of motion describing dynamics of a serial robot consisting of N rigid bodies can be written in a well known form as

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{c}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q}) + \mathbf{Q}_R(\dot{\mathbf{q}}) = \mathbf{Q}_M, \tag{1}$$

where $\mathbf{q} \in \mathbb{R}^N$ denotes vector of generalized coordinates, dot over the symbol stands for the derivative with respect to time, $\mathbf{M}(\mathbf{q}) \in \mathbb{R}^{N,N}$ denotes symmetric positive definite mass matrix, $\mathbf{c}(\mathbf{q},\dot{\mathbf{q}}) \in \mathbb{R}^N$ stands for vector of centripetal and Coriolis terms, and $\mathbf{g}(\mathbf{q}) \in \mathbb{R}^N$ denotes gravity vector. Furthermore, vector $\mathbf{Q}_R \in \mathbb{R}^N$ stands for friction forces while $\mathbf{Q}_M \in \mathbb{R}^N$ denotes torques acting on bodies, i.e active or control torques.

B. Friction model

Dissipative forces is the model are assumed in the form of Coulomb's dry and viscous friction, leading to

$$Q_{R_i} = r_{v_i}\dot{q}_i + r_{c_i} sign(\dot{q}_i) \;, \quad i = 1 \dots N, \tag{2}$$

where r_{v_i} and r_{c_i} are respectively coefficients of viscous and dry friction. In order to avoid non-smooth function in the model, sign function is approximated with tangent hyperbolic function as

$$sign(\dot{q}_i) \approx \tanh(\frac{\dot{q}_i}{\varepsilon}), \tag{3}$$

where ε is very small number chosen to make slope of the tangent hyperbolic function very steep around zero.

C. Motor and gearbox dynamics

Assuming that at each joint, a motor and a gearbox are located leads to motor dynamics in the form

$$i_{G,i}^2 C_{M,i}\ddot{q}_i = i_{G,i} M_{Mot,i} = Q_{M,i}, \quad i = 1 \dots N, \tag{4}$$

where, $C_{M,i}$ stand for the rotors axial moment of inertia corresponding to rotation axis while $M_{Mot,i}$ stands for the motor torque. Note that the previous equations can be divided by $i_{G,i}$, however between a motor and a body is the gearbox, thus torque $Q_{M,i}$, acting on body i, is $i_{G,i}$ times greater than the motor torque. Also, note that although rotor in a motor rotates around an axis that is itself in motion and thus making rotors motion complex in the parallel sense, dynamics of a motor and gearbox is taken in a much simplified form. Namely, assuming known gear ratio $i_{G,i}$, rotor of a motor driving body i spins around the joint axis with angular velocity $i_{G,i}$ times greater than relative angular velocity of the corresponding bodies. Since this rotation is dominant compared to the motion of the joint axis itself, only it is taken into account.

III. METHODOLOGY FOR IDENTIFICATION OF DYNAMICAL PARAMETERS

Methodology for identification of robot parameters is based of the fact that the equations describing motion of a system of rigid bodies can be written in linear form with respect to some set of dynamical parameters, see [2], [3]. For an overview on robot dynamic parameter identification see [10].

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A. Parameter linear form of the equations of motion

Having the previous in mind, (1) is written as

$$\sum_{i=1}^{N}\Theta_{Ti}(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})\mathbf{p}_{Ti} = \Theta_T(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})\mathbf{p}_T = \mathbf{Q}^o, \quad \Theta_T \in \mathbb{R}^{N,10N},\ \mathbf{p}_T \in \mathbb{R}^{10N}, \tag{5}$$

where

$$\Theta_{Ti} = \mathbf{F}_{K_i}^T \times \left[\begin{array}{c|c|c} (\dot{\mathbf{v}}_K + \tilde{\omega}\mathbf{v}_K - \mathbf{g}) & (\dot{\tilde{\omega}} + \tilde{\omega}\tilde{\omega}) & 0 \\ \hline 0 & -(\dot{\mathbf{v}}_K + \tilde{\omega}\mathbf{v}_K - \mathbf{g})^{\sim} & (\dot{\Omega} + \tilde{\omega}\Omega \mid -\dot{\hat{\Omega}} - \tilde{\omega}\hat{\Omega}) \end{array}\right],$$

$$\mathbf{F}_{K_i} = \left[(\frac{\partial_K \mathbf{v}_K}{\partial \dot{\mathbf{q}}})^T \quad (\frac{\partial_K \omega_{IK}}{\partial \dot{\mathbf{q}}})^T \right]_i^T \in \mathbb{R}^{6,N}. \tag{6}$$

For the derivation of the previous equation see [7].

Parameter vector $\mathbf{p}_{Ti}$ is

$$\mathbf{p}_{Ti} = \left(m, m\rho_{Sx}, m\rho_{Sy}, m\rho_{Sz}, A, B, C, D, E, F\right)_i^T \in \mathbb{R}^{10}, \tag{7}$$

where ρ_{Sx}, ρ_{Sy} and ρ_{Sz} are projections of the center of mass of body i onto x, y and z axes of the coordinate frame positioned, and rigidly connected, to the joint of that body and whose one axis is the rotation axis of that body. In the same coordinate system, moments of inertia of i-th body are denoted as A, B, C, D, E, F. Furthermore, in (6) matrices Ω and $\hat{\Omega}$ stand for

$$\mathbf{J}_{K\,K}\omega_{IK} = \begin{bmatrix} A & -F & -E \\ -F & B & -D \\ -E & -D & C \end{bmatrix} \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \left[\Omega \mid \hat{\Omega}\right] \begin{pmatrix} A \\ B \\ C \\ D \\ E \\ F \end{pmatrix}, \tag{8}$$

$$\Omega = \begin{bmatrix} \omega_x & 0 & 0 \\ 0 & \omega_y & 0 \\ 0 & 0 & \omega_z \end{bmatrix}, \quad \hat{\Omega} = \begin{bmatrix} 0 & \omega_z & \omega_y \\ \omega_z & 0 & \omega_x \\ \omega_y & \omega_x & 0 \end{bmatrix}, \tag{9}$$

vector $\mathbf{g}$ denotes acceleration vector of gravity, and $\tilde{(\cdot)}$ is a skew-symmetric matrix corresponding to a vector $(\cdot)$. Note that the inertia matrix $\mathbf{J}_K$ and all vectors in (6) are written in the body coordinate frames positioned at joints. Vector $\mathbf{Q}^o$ in (5), in the absence of motor dynamics and friction, denotes vector of body torques, while for the case of friction and motor dynamics is defined in what follows.

B. Parameter linear form of the motor dynamics and friction forces

Differential equations (4) describing motor dynamics is written in parameter linear form as

$$\mathbf{Q}_M = \left[diag(\ddot{q}_i)\right] \begin{pmatrix} i_{G,1}^2 C_{M,1} \\ \vdots \\ i_{G,N}^2 C_{M,N} \end{pmatrix} = \Theta_{TM}\,\mathbf{p}_{TM}, \tag{10}$$

where vector of parameters is

$$\mathbf{p}_{TM} = \begin{pmatrix} i_{G,1}^2 C_{M,1} \\ \vdots \\ i_{G,N}^2 C_{M,N} \end{pmatrix}. \tag{11}$$

Dissipative forces defined in (2) are written in parameter linear form as

$$\mathbf{Q}_R = \left[\ diag(\dot{q}_i) \mid diag(sign(\dot{q}_i))\ \right] \begin{pmatrix} r_{v_1} \\ \vdots \\ r_{c_N} \end{pmatrix} = \Theta_R\,\mathbf{p}_R, \tag{12}$$

where $diag(\cdot)$ denotes diagonal matrix, and where parameter vector is

$$\mathbf{p}_R = \begin{pmatrix} r_{v_1} \\ \vdots \\ r_{c_N} \end{pmatrix}. \tag{13}$$

C. Parameter linear form of the equations describing the whole system

When equations describing all element of the model, i.e. rigid bodies, motors and friction, are written in parameter linear form, writing the same form of equations describing the system in whole is very easy. Namely, combining (5), (10) and (12), linear form of equations describing the whole system is

$$\left[\ \Theta_T \quad \Theta_{TM} \quad \Theta_R\ \right] \begin{pmatrix} \mathbf{p}_T \\ \mathbf{p}_{TM} \\ \mathbf{p}_R \end{pmatrix} = \Theta\,\mathbf{p} = \mathbf{Q}_M, \quad \Theta \in \mathbb{R}^{N,13N},\ \mathbf{p} \in \mathbb{R}^{13N},\ \mathbf{Q}_M \in \mathbb{R}^N, \tag{14}$$

where matrix Θ is known as the regressor matrix of the system. From the previous equations vector $\mathbf{Q}^o$ from (5) is

$$\mathbf{Q}^o = \mathbf{Q}_M - \Theta_R\,\mathbf{p}_R - \Theta_{TM}\mathbf{p}_{TM}. \tag{15}$$

D. Determination of the base parameters

Before determination of the base parameters, zero columns in the regressor matrix are identified and eliminated. Namely, in the regressor matrix defined in (5), the most general type of rigid body motion, i.e. translation plus rotation, is assumed for every body in the kinematic chain. However, when it comes to robot manipulators, the motion of the first segment in chain can be described as pure rotation around an axis. Thus, only columns in the regressor corresponding to the moments of inertia related to the axis of rotation in parameter vector (7) are not equal to zero. All other columns in the regressor matrix for the first body in chain are equal to zero. Note that if the coordinate frame, located at joint axis of the second body in chain, is positioned in such a way that the velocity of its origin is always equal to zero, then the projection of the center of mass of that body, on the axis of rotation can not be identified. However, this can be easily avoided by moving that frame along the axis of rotation.

Computation of the base parameters is based on determination of independent columns of the regressor matrix Θ by use of the QR decomposition. This procedure is explained in details in [5], Appendix 5. Here it is assumed that the base parameters and the corresponding independent columns are determined. Thus, (14) can be written as

$$\Theta(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})\,\mathbf{p} = \Theta_B(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})\,\mathbf{p}_B = \mathbf{Q}_M, \quad \Theta_B \in \mathbb{R}^{N,b},\ \mathbf{p}_B \in \mathbb{R}^b,\ \mathbf{Q}_M \in \mathbb{R}^N \tag{16}$$

where Θ_B is the new regressor with all columns being independent, and where $\mathbf{p}_B$ is the vector of base parameters. Note that the system of equations (45), with N equations and $13N$ unknowns, is undetermined and thus have none or infinitely many solution. Also note that the elimination of zero columns is not necessary because when calculating the base parameters, the parameters corresponding to zero columns are not present. However, elimination of zero columns is the standard procedure in the determination of base parameters.

In order to determine the base parameters (16), the real system is excited with specially chosen excitation trajectory and the generalized coordinates and the motor torques are measured at m time instances. From the generalized coordinates, the generalized velocities and accelerations are calculated using filtering and than the new regressor, called information matrix, is formed as

$$\begin{pmatrix} \Theta_B|_{t_1} \\ \vdots \\ \Theta_B|_{t_m} \end{pmatrix} \mathbf{p}_B = \begin{pmatrix} \mathbf{Q}_M|_{t_1} \\ \vdots \\ \mathbf{Q}_M|_{t_m} \end{pmatrix} + \mathbf{r}_n, \tag{17}$$

or written in a simpler form as

$$\overline{\Theta}_B \mathbf{p}_B = \overline{\mathbf{Q}}_M + \mathbf{r}_n, \tag{18}$$

where $\mathbf{r}_n$ is the residual error vector. Now instead of an undetermined, an over determined system of equations is obtained. This system usually does not admit a solution or it can be found only for some special cases. However, an approximate solution of the problem can be found by solving least squares problem

$$\min_{\mathbf{p}_B} \| \frac{1}{2} \mathbf{e}^T \mathbf{e} \|, \quad \mathbf{e} = \overline{\Theta}_B \mathbf{p}_B - \overline{\mathbf{Q}}_M. \tag{19}$$

where the solution is

$$\begin{aligned} \left\{ \frac{\partial}{\partial \mathbf{p}} \left[\frac{\mathbf{e}^T \mathbf{e}}{2} \right] \right\}^T &= \overline{\Theta}_B^T \overline{\Theta}_B \mathbf{p}_B - \overline{\Theta}_B^T \overline{\mathbf{Q}}_M = 0 \\ \Longrightarrow \mathbf{p}_B &= \left[\overline{\Theta}_B^T \overline{\Theta}_B \right]^{-1} \overline{\Theta}_B^T \overline{\mathbf{Q}}_M, \end{aligned} \tag{20}$$

provided that the matrix $\left[\overline{\Theta}_B^T \overline{\Theta}_B \right]^{-1}$ exists, i.e. if $\overline{\Theta}_B$ has full column rank. Since the matrix $\overline{\Theta}_B$ has linearly independent columns it is a full rank matrix. Note that the matrix $\left[\overline{\Theta}_B^T \overline{\Theta}_B \right]^{-1} \overline{\Theta}_B^T$ is a pseudo inverse of the matrix $\overline{\Theta}_B$, or more precisely the left Moore-Penrose inverse. Instead of using the pseudo inverse, the minimization problem (19) can also be solved using direct numerical optimization.

Assuming that the matrix $\overline{\Theta}_B$ is deterministic and that ρ_n is zero mean additive independent noise, the standard deviation of the i -th parameter is, see [5],

$$\sigma_i = \sqrt{ (\left[\overline{\Theta}_B^T \overline{\Theta}_B \right]^{-1})_{i,i} }. \tag{21}$$

If the standard deviation of a parameter is big, then parameter is considered to be poorly identified.

In order to quantify how good calculated base parameters predict measured torques, normalized error

$$e_N = \frac{1}{m} \sqrt{\mathbf{e}^T \mathbf{e}}, \tag{22}$$

is used, where m stands for the number of time samples used for the calculation of the information matrix. Assuming that all degrees of freedom are rotational, the unit of this error is the newton meter.

Here, it is important to note that (19) and (22) have sense only is all degrees of freedom are of the same type, e.g. rotational. Otherwise, dimensionless quantities must be introduced first and only then (19) and (22) have sense.

Finally, note that the good approximate solution of Eq. (19) can be found only if the excitation trajectory excites all dynamical parameters of the robot. Determination of such trajectory is the subject of the next subsection.

E. Determination of the identification trajectory

The identification trajectory that excites all dynamic parameters, and thus yields good approximate solution for the parameter identification problem (19), is usually called the persistent excitation trajectory. The term ”persistent” means that all parameters must be excited persistently throughout time, that is, on every time interval during the identification process. There are various criteria for calculating persistent excitation, see [8], [1], [4]. However, one of the most used is the condition number of the matrix $\Lambda = \overline{\Theta}_B^T \overline{\Theta}_B$ because it measures the sensitivity of the solution of the least squares problem to the modeling errors and noise. Thus, ”good” excitation trajectory is the one whose points in time give a small condition number of the matrix Λ. Several condition number based criteria for calculating the persistent excitation exist in the literature, see [5]. Here, the criteria

$$\min_{\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}} \; cond(\Lambda(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})) = \frac{\sigma_{\max}}{\sigma_{\min}} \geq 1 \tag{23}$$

is used where $\sigma_{\max}$ and $\sigma_{\min}$ denote the maximum and the minimum singular value of the matrix Λ, respectively. Since real physical robot cannot achieve arbitrary values of coordinates, velocities and accelerations, the previous minimization problem is solved together with constrains

$$\begin{aligned} \mathbf{q}_{\min} \leq \mathbf{q} &\leq \mathbf{q}_{\max}, \\ |\dot{\mathbf{q}}| &\leq \dot{\mathbf{q}}_{\max}, \\ |\ddot{\mathbf{q}}| &\leq \ddot{\mathbf{q}}_{\max}, \end{aligned} \tag{24}$$

where the vectors $\mathbf{q}_{\min}$ and $\mathbf{q}_{\max}$ denote minimal and maximal allowed values of the generalized coordinates, the vector $\dot{\mathbf{q}}_{\max}$ stands for maximal generalized velocities and the vector $\ddot{\mathbf{q}}_{\max}$ denotes maximal allowed generalized accelerations. If the robot can self collide during motion, than also the requirement that there is no self collision is used as the constraint. Besides condition number, determinant of the matrix Λ can also be used for calculating persistent excitation, see [4].

In order to solve the minimization problem (23) together with constrains (24), following [9] the minimization trajectory will be taken in form of a finite Fourier series as

$$q_i(t) = \sum_{l=1}^{L_i} \left(\frac{a_{i,l}}{\omega_i l} \sin(\omega_i l t) - \frac{b_{i,l}}{\omega_i l} \cos(\omega_i l t) \right) + q_{i,0}, \tag{25}$$

where L_i is the order of the series, ω_i is the base frequency, $q_{i,0}$ is the coordinate offset, and $a_{i,l}$ and $b_{i,l}$ are coefficients of the series. In the general case, all constants in the previous equation can be used as optimization variables. However, usually the order of the series is fixed and the rest variables are used in optimization. With the Fourier series representation the infinite-dimensional optimization problem (23) is substituted with finite dimensional one given as

$$\min_{\mathbf{a},\mathbf{b},\omega,\mathbf{q}_0} \quad cond(\Lambda(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})) \tag{26}$$

where

$$\begin{aligned}\mathbf{a} &= \left[a_{1,1}\ldots a_{1,L_i}\ldots a_{N,L_i}\right]^T\\ \mathbf{b} &= \left[b_{1,1}\ldots b_{1,L_i}\ldots b_{N,L_i}\right]^T\\ \omega &= \left[\omega_1\ldots\omega_N\right]^T\\ \mathbf{q}_0 &= \left[q_{1,0}\cdots q_{N,0}\right]^T,\end{aligned} \tag{27}$$

which is again solved together with the constrains (24) and the condition that there is no self collision of the robot. Finally, instead of optimizing all previously mentioned variables, for example the coordinate offset $\mathbf{q}_0$ can be predefined or the basic frequency ω_i can be the same for all bodies. This lowers the dimension of solution of the problem and thus also the time needed for optimization algorithm to find the solution.

IV. Universal robots UR5

As an example for demonstrating the previously described methodology for parameter identification, the Universal Robots UR5 manipulator is used, see Fig. 1. This manipulator has six degrees of freedom and is a lightweight collaborative robot.

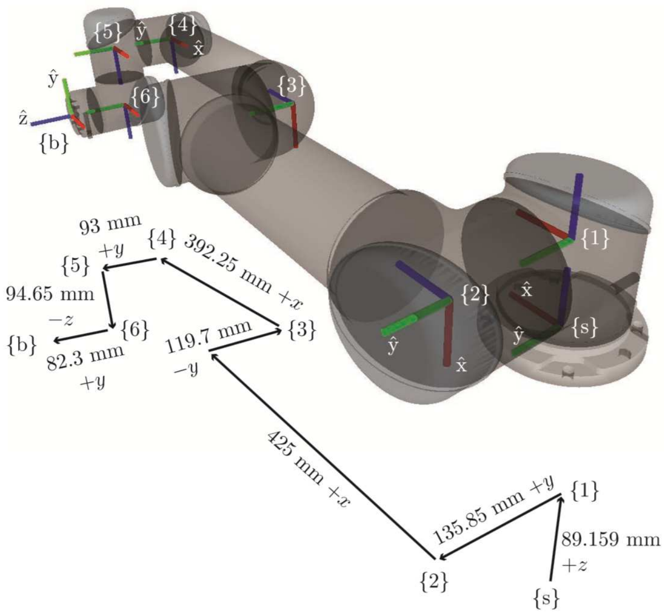

Fig. 1. Universal Robots UR5, taken from [6].

In Fig. 1, UR5 robot is shown at initial configuration, together with the coordinate systems of interest and distances between them. In what follows, the relative position and orientation of these coordinate frames at initial configuration are described. Furthermore, all generalized coordinates to be introduced are measured in positive mathematical directions, relative to the previous body in the kinematic chain, and given in radians.

Inertial frame of reference is denoted as K_sxyz, where x, y and z axis are shown as red, green and blue axis, respectively. Translating the inertial frame along its z axis, for value $l_{1,z}$, coordinate frame K_1xyz attached to the first segment is obtained. The orientation of these two coordinate frames is the same at initial configuration, while an arbitrary orientation of the first body is achieved by rotating it in positive mathematical direction around K_1z axis, where the generalized coordinate describing this rotation is denoted as $q_1 = q_1(t)$. Further, coordinate frame K_2xyz is obtained by translating the frame K_1xyz along K_1y axis, for value $l_{2,y}$, and then by rotating it in the positive mathematical direction around the same axis for angle $\pi/2$. An arbitrary orientation of the second body, with respect to the first, is obtained by rotation around K_2y axis for angle $q_2 = q_2(t)$. Next, translating the coordinate frame K_2xyz along K_2y and K_2z axis for values $l_{3,y}$ and $l_{3,z}$, respectively, coordinate frame K_3xyz is obtained. Rotating the third body around K_3z axis, again in the positive mathematical direction, its arbitrary orientation is achieved. This rotation is described with generalized coordinate $q_3 = q_3(t)$. The coordinate frame K_4xyz is obtained by translating frame K_3xyz along K_3z axis for value $l_{4,z}$ and by rotating it in the positive mathematical direction around K_3y axis for angle $\pi/2$. An arbitrary orientation of the fourth body is achieved by rotation around K_4y for angle $q_4 = q_4(t)$. For obtaining coordinate frame K_5xyz attached to the fifth body in the kinematic chain, coordinate frame K_4xyz is translated along the K_4y axis for value $l_{5,y}$. By rotating the fifth body around K_5z axis, its arbitrary orientation is obtained, where the generalized coordinate describing that rotation is $q_5 = q_5(t)$. Translating the coordinate frame K_5xyz along K_5z axis for value $l_{6,z}$, coordinate frame K_6xyz is obtained. An arbitrary orientation of the sixths body is achieved by rotation around K_6y axis, and that rotation is described by the generalized coordinate $q_6 = q_6(t)$. Finally, the coordinate frame K_7xyz, positioned at the end effector is obtained by translating frame K_6xyz along K_6y for value $l_{7,y}$ and than rotating it in the negative mathematical direction for angle $\pi/2$.

With the previously introduced generalized coordinates, the orthogonal transformation matrices are

$$\begin{aligned}
\mathbf{R}_{K_s,K_1} &= \mathbf{R}_z(q_1), \\
\mathbf{R}_{K_1,K_2} &= \mathbf{R}_y(\frac{\pi}{2})\mathbf{R}_y(q_2) = \mathbf{R}_y(\frac{\pi}{2}+q_2), \\
\mathbf{R}_{K_2,K_3} &= \mathbf{R}_y(q_3), \\
\mathbf{R}_{K_3,K_4} &= \mathbf{R}_y(\frac{\pi}{2})\mathbf{R}_y(q_4) = \mathbf{R}_y(\frac{\pi}{2}+q_4), \\
\mathbf{R}_{K_4,K_5} &= \mathbf{R}_z(q_5), \\
\mathbf{R}_{K_5,K_6} &= \mathbf{R}_y(q_6), \\
\mathbf{R}_{K_6,K_7} &= \mathbf{R}_x(-\frac{\pi}{2}),
\end{aligned} \tag{28}$$

where the rotation matrices corresponding to rotation around x, y and z axis, are

$$\mathbf{R}_x(q_i) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(q_i) & -\sin(q_i) \\ 0 & \sin(q_i) & \cos(q_i) \end{pmatrix}, \tag{29}$$

$$\mathbf{R}_y(q_i) = \begin{pmatrix} \cos(q_i) & 0 & sin(q_i) \\ 0 & 1 & 0 \\ -\sin(q_i) & 0 & \cos(q_i) \end{pmatrix} \tag{30}$$

and

$$\mathbf{R}_z(q_i) = \begin{pmatrix} \cos(q_i) & -sin(q_i) & 0 \\ \sin(q_i) & \cos(q_i) & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{31}$$

A. Kinematic analysis

Starting from the angular velocity of the first body and following the recursive formulation, angular velocities of the bodies in the kinematic chain are

$${}_{K_i}\omega_{I,K_i} = \mathbf{R}_{K_i,K_{i-1}}\ {}_{K_{i-1}}\omega_{I,K_{i-1}} + {}_{K_i}\omega_{K_{i-1},K_i}\ , \quad i = 2\dots6, \tag{32}$$

where

$$\begin{aligned}
{}_{K_1}\omega_{I,K_1} &= \begin{bmatrix}0 & 0 & \dot{q}_1\end{bmatrix}^T, \\
{}_{K_2}\omega_{K_1,K_2} &= \begin{bmatrix}0 & \dot{q}_2 & 0\end{bmatrix}^T, \\
{}_{K_3}\omega_{K_2,K_3} &= \begin{bmatrix}0 & \dot{q}_3 & 0\end{bmatrix}^T, \\
{}_{K_4}\omega_{K_3,K_4} &= \begin{bmatrix}0 & \dot{q}_4 & 0\end{bmatrix}^T, \\
{}_{K_5}\omega_{K_4,K_5} &= \begin{bmatrix}0 & 0 & \dot{q}_5\end{bmatrix}^T, \\
{}_{K_6}\omega_{K_5,K_6} &= \begin{bmatrix}0 & \dot{q}_6 & 0\end{bmatrix}^T.
\end{aligned} \tag{33}$$

With the angular velocities defined, velocity of the origin of frame K_ixyz, written in that frame, is

$${}_{K_i}\mathbf{v}_{K_i} = \mathbf{R}_{K_i,K_{i-1}}\ {}_{K_{i-1}}\mathbf{v}_{K_i}, \quad i = 1\dots6 \tag{34}$$

where

$$\begin{aligned}
&{}_{K_0}\mathbf{v}_{K_1} = \mathbf{0}, \\
&{}_{K_i}\mathbf{v}_{K_{i+1}} = \mathbf{R}_{K_i,K_{i-1}}\ {}_{K_{i-1}}\mathbf{v}_{K_i} + {}_{K_i}\tilde{\omega}_{I,K_i}\ {}_{K_i}\mathbf{r}_{K_i,K_{i+1}}\ , \quad i = 1\dots5;
\end{aligned} \tag{35}$$

and where the index 0 denotes inertial frame of reference. In the previous equations, vectors ${}_{K_i}r_{K_i,K_{i+1}}$, $i = 1\dots6$ are

$$\begin{aligned}
{}_{K_1}r_{K_1,K_2} &= \begin{bmatrix}0 & l_{2,y} & 0\end{bmatrix}^T, \\
{}_{K_2}r_{K_2,K_3} &= \begin{bmatrix}0 & l_{3,y} & l_{3,z}\end{bmatrix}^T, \\
{}_{K_3}r_{K_3,K_4} &= \begin{bmatrix}0 & 0 & l_{4,z}\end{bmatrix}^T, \\
{}_{K_4}r_{K_4,K_5} &= \begin{bmatrix}0 & l_{5,y} & 0\end{bmatrix}^T, \\
{}_{K_5}r_{K_5,K_6} &= \begin{bmatrix}0 & 0 & l_{6,z}\end{bmatrix}^T.
\end{aligned} \tag{36}$$

Differentiating with respect to time (32), vectors of angular accelerations of bodies are obtained. Similarly, differentiating with respect to time (34), and taking into account both change of intensity and of direction, acceleration vectors of points K_i, $i = 1\dots6$ are obtained as

$${}_{K_i}\mathbf{a}_{K_i} = {}_{K_i}\dot{\mathbf{v}}_{K_i} + {}_{K_i}\tilde{\omega}_{I,K_iK_i}\mathbf{v}_{K_i}. \tag{37}$$

B. Parameter linear form of the equations of motion

Since all elements for writing the parameter linear form of the equations of motion are known, in order to construct the regressor matrix, it is necessary to substitute them into (6), (10), and (12). However, the obtained analytical expression for the regressor is not shown. Instead, it will be assumed that the regressor matrix $\Theta(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})$ is known. Then, the parameter linear form of the equations of motion is

$$\Theta(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})\,\mathbf{p} = \mathbf{Q}_M, \quad \Theta \in \mathbb{R}^{6,78}\ ,\ \mathbf{p} \in \mathbb{R}^{78},\ \mathbf{Q}_M \in \mathbb{R}^6. \tag{38}$$

where the parameter vector $\mathbf{p}$ is

$$\mathbf{p} = \begin{pmatrix} \mathbf{p}_T \\ \mathbf{p}_{TM} \\ \mathbf{p}_R \end{pmatrix} \in \mathbb{R}^{78}, \tag{39}$$

with its elements defined as

$$\begin{aligned}
\mathbf{p}_T &= \left(\mathbf{p}_{T1}\dots\mathbf{p}_{T6}\right) \in \mathbb{R}^{60}, \\
\mathbf{p}_{Ti} &= \left(m, m\rho_{Sx}, m\rho_{Sy}, m\rho_{Sz}, A, B, C, D, E, F\right)_i \in \mathbb{R}^{10}, \quad i = 1\dots6, \\
\mathbf{p}_{TM} &= \left(i_{G,1}^2 C_{M,1}\dots i_{G,6}^2 C_{M,6}\right) \in \mathbb{R}^6, \\
\mathbf{p}_R &= \left(r_{v_1}\dots r_{v_6}\ r_{c_1}\dots r_{c_6}\right) \in \mathbb{R}^6,
\end{aligned} \tag{40}$$

where ρ_{Sx_i}, ρ_{Sy_i} and ρ_{Sz_i} are projections of the center of mass of body i onto the axis of the coordinate frame K_ixyz, respectively, i.e.

$${}_{K_i}r_{K_i,S_i} = \begin{bmatrix}\rho_{Sx_i} & \rho_{Sy_i} & \rho_{Sz_i}\end{bmatrix}^T, \quad i = 1\dots6. \tag{41}$$

Note that, since motion of the first body is described as pure rotation, only a column in the matrix $\Theta(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})$, corresponding to axial moments of inertia for the axis of rotation is not zero. All other columns in that regressor are zero.

Substituting random values for vectors $\mathbf{q}$, $\dot{\mathbf{q}}$ and $\ddot{\mathbf{q}}$ in the matrix $\Theta(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})$, and applying the QR decomposition to the resulting matrix, results in the base parameter vector

$$
\mathbf{p}_B = \left(\begin{array}{c}
i_{G,1}^2\, C_{M,1} + C_1 + C_2 + C_3 + C_4 + \\
+0.01285\, m_2 + 0.01191\, m_5 + \\
+0.01191\, m_6 + 0.2267\, m_2\, \rho_{Sy_2} \\
\hline
m_2\, \rho_{Sx_2} \\
\hline
0.425\, m_3 + 0.425\, m_4 + 0.425\, m_5 + \\
+0.425\, m_6 + m_2\, \rho_{Sz_2} \\
\hline
A_2 - C_2 + 0.1806\, m_3 + 0.1806\, m_4 + \\
+0.1806\, m_5 + 0.1806\, m_6 \\
\hline
i_{G,2}^2\, C_{M,2} + B_2 + 0.1806\, m_3 + 0.1806\, m_4 + \\
+0.1806\, m_5 + 0.1806\, m_6 \\
\hline
D_2 - 0.04818\, m_3 - 0.04818\, m_4 - \\
-0.001789\, m_5 - 0.001789\, m_6 + \\
+0.425\, m_3\, \rho_{Sy_3} + 0.425\, m_4\, \rho_{Sy_4} \\
\hline
E_2 \\
F_2 \\
m_3\, \rho_{Sx_3} \\
0.3922\, m_4 + 0.3922\, m_5 + 0.3922\, m_6 + m_3\, \rho_{Sz_3} \\
A_3 - C_3 + 0.1539\, m_4 + 0.1539\, m_5 + 0.1539\, m_6 \\
B_3 + 0.1539\, m_4 + 0.1539\, m_5 + 0.1539\, m_6 \\
D_3 + 0.04281\, m_5 + 0.04281\, m_6 + 0.3922\, m_4\, \rho_{Sy_4} \\
E_3 \\
F_3 \\
m_4\, \rho_{Sx_4} \\
0.09465\, m_6 + m_4\, \rho_{Sz_4} + m_5\, \rho_{Sz_5} \\
A_4 + B_5 - C_4 + 0.008959\, m_6 \\
B_4 + B_5 + 0.008959\, m_6 \\
D_4 + 0.01033\, m_6 + 0.1092\, m_5\, \rho_{Sz_5} \\
E_4 \\
F_4 \\
m_5\, \rho_{Sx_5} \\
m_5\, \rho_{Sy_5} + m_6\, \rho_{Sy_6} \\
A_5 - B_5 + C_6 \\
C_5 + C_6 \\
D_5 + 0.09465\, m_6\, \rho_{Sy_6} \\
E_5 \\
F_5 \\
m_6\, \rho_{Sx_6} \\
m_6\, \rho_{Sz_6} \\
A_6 - C_6 \\
B_6 \\
D_6 \\
E_6 \\
F_6 \\
\left[i_{G,3}^2\, C_{M,3} \ldots i_{G,6}^2\, C_{M,6} \right]^T \\
\left[r_{v_1} \ldots r_{c_6} \right]^T
\end{array}\right). \tag{42}
$$

Thus, the system of equations

$$
\Theta(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})\, \mathbf{p} = \mathbf{Q}_M, \quad \Theta \in \mathbb{R}^{6,78},\ \mathbf{p} \in \mathbb{R}^{78},\ \mathbf{Q}_M \in \mathbb{R}^6 \tag{43}
$$

is substituted with the new system

$$
\Theta_B(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})\, \mathbf{p}_B = \mathbf{Q}_M, \quad \Theta_B \in \mathbb{R}^{6,52},\ \mathbf{p}_B \in \mathbb{R}^{52},\ \mathbf{Q}_M \in \mathbb{R}^6 \tag{44}
$$

where all columns in the new regressor Θ_B are mutually independent. Note that elements of the vector $\mathbf{p}_B$ are linear combination of the model parameters. Also note that the zero columns from the regressor are not eliminated first, but the corresponding parameters are still not in the vector $\mathbf{p}_B$. They are eliminated by use of the QR decomposition.

Sometimes, some parameters are known to be zero or they are negligible compared to some other parameters. In that case one can chose not to identify them so the corresponding columns in matrix Θ are eliminated first and then the QR decomposition is applied to the resulting matrix. This results in a new base parameter vector.

In this work, for the identification of parameters of the UR5 manipulator, several parameters are assumed to be negligible. Namely, centrifugal moments of inertia of links are assumed to be much smaller that the axial moments of inertia and thus are not going to be identified. Furthermore, it is assumed that position of the center of mass of body i does not have all three projections on the axis of the coordinate frames $K_i xyz, i = 1 \ldots 6$, but only one. The motion of the first body in the kinematic chain is pure rotation and thus only the axial moment of inertia corresponding to the rotation axis is identified. For the second body, it is assumed that the center of mass has projection only on the $K_2 z$ axis. Similarly, the center of mass of the third body is assumed to be on $K_3 z$ axis. For the forth and the sixth body in chain, it is assumed that the corresponding centers of mass are on $K_4 y$ and $K_6 y$ axis, respectively. Finally, for the fifth body, the center of mass is assumed to lie on the $K_5 z$ axis.

With the previous assumptions, the base parameter vector $\mathbf{p}_B \in \mathbb{R}^{33}$ is now

$$
\mathbf{p}_B = \left(\begin{array}{c}
i_{G,1}^2\, C_{M,1} + C_1 + C_2 + C_3 + C_4 + 0.01285\, m_2 + \\
+0.01191\, m_5 + 0.01191\, m_6 \\
\hline
m_2\, \rho_{Sz_2} \\
A_2 - C_2 \\
i_{G,2}^2\, C_{M,2} + B_2 \\
m_3 + m_4 + m_5 + m_6 \\
0.3922\, m_4 + 0.3922\, m_5 + 0.3922\, m_6 + m_3\, \rho_{Sz_3} \\
A_3 - C_3 + 0.1539\, m_4 + 0.1539\, m_5 + 0.1539\, m_6 \\
B_3 + 0.1539\, m_4 + 0.1539\, m_5 + 0.1539\, m_6 \\
0.1092\, m_5 + 0.1092\, m_6 + m_4\, \rho_{Sy_4} \\
A_4 + B_5 - C_4 + 0.008959\, m_6 \\
B_4 + B_5 + 0.008959\, m_6 \\
0.09465\, m_6 + m_5\, \rho_{Sz_5} \\
A_5 - B_5 + C_6 \\
C_5 + C_6 \\
m_6\, \rho_{Sy_6} \\
A_6 - C_6 \\
B_6 \\
i_{G,3}^2\, C_{M,3} \\
i_{G,4}^2\, C_{M,4} \\
i_{G,5}^2\, C_{M,5} \\
i_{G,6}^2\, C_{M,6} \\
r_{v_1} \\
\vdots \\
r_{c_6}
\end{array}\right). \tag{45}
$$

Note that, if some parameters are not going to be identified, the new base parameter vector is not obtained by simply substituting zeros for those parameters in the vector shown in (42). In what follows the base parameter vector (45) is going to be identified.

C. *Identification results*

For the identification of the base parameters, two persistent excitation trajectories are generated. One is used for parameter identification and the other one for validation of the obtained parameter vector. These trajectories are generated by solving the optimization problem (26), where the order of the series in (25) is 5, and where the offset $q_{2,0} = -\pi/2$ and all others are zero. The rest parameters in Fourier series are found by optimization. The identification is done on a time interval of 20 seconds, however, only first 10 seconds are shown in figures. In Fig. 2 measured angles of the excitation used for the parameter identification are shown, while Fig. 3 shows measured motor currents for the same trajectory.

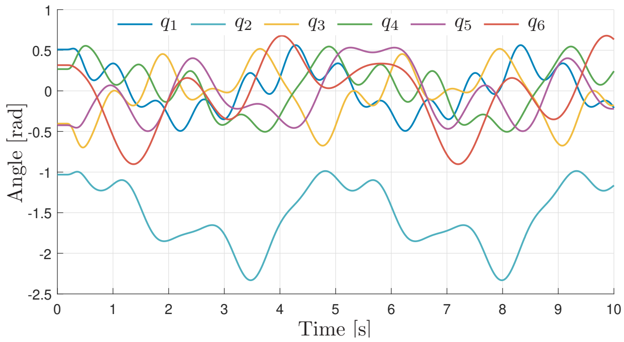

Fig. 2. Persistent excitation trajectories used for the identification

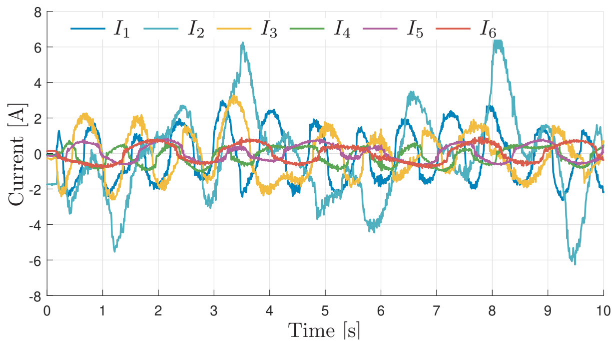

Fig. 3. Motor current

In order to calculate torques acting on bodies, each motor current is multiplied with torques constant and by gear ratio. Thus, body torques are $M_i = i_{G,i} k_i I_i$, $i = 1 \dots 6$. On the UR5 robot, there are two types of motors, one with motor constant $k_i = 0.125$ Nm/A, $i = 1 \dots 3$, and other with constant $k_i = 0.0922$ Nm/A, $i = 4 \dots 6$. Also, all gears have the same gear ration, i.e. $i_G = i_{G,i} = 101$, $i = 1 \dots 6$.

In order to form the regressor Θ_B, generalized velocities and accelerations must be calculated from the measured values of generalized coordinates. When working with the UR5 robot, generalized velocities are obtained from the controller, while generalized accelerations are calculated using filtering. The transfer function of the filter used is

$$y = \frac{s}{\frac{s}{w} + 1} u, \tag{46}$$

where s denotes the Laplace variable, $w = 2\pi f$ is angular frequency with $f = 10$ Hz being the corner frequency of the filter. The values of the corner frequency is determined by inspecting the frequency content of the measured signals. Using the filter and Matlab's "*filtfilt*" function, generalized acceleration are obtained. With the previous preparation done, one can proceed to the determination of the base parameters as described in Section III.

Following the methodology for the parameter identification, first the information matrix $\overline{\Theta}_B$ and vector $\overline{\mathbf{Q}}_M$ are formed. Then, base parameter are determined in two ways. First by using Moore-Penrose pseudoinverse from (20), and then by using numerical optimization to directly solve the optimization problem (19), together with the constraint that all base parameters are positive.

The results for the base parameters obtained by use of the pseudoinverse are shown in Fig. 4, together with the corresponding standard deviations. In Fig. 5, base parameters obtained by optimization are shown, again with the corresponding standard deviations.

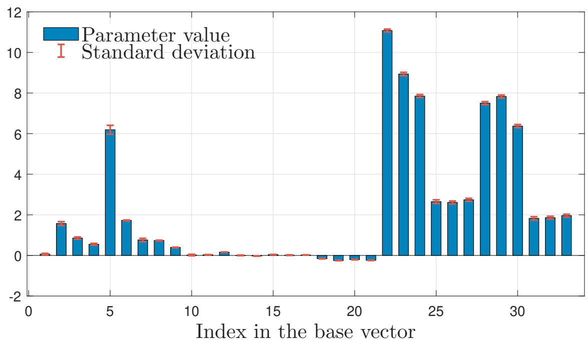

Fig. 4. Base parameters obtained by pseudoinverse

Note that the standard deviations are small, and the same in both figures.

In order to check the quality of the calculated base parameters vector, predicted torques are compared with the measured ones and the normalized error (22) is calculated. Predicted body torques, obtained using the base parameters vector obtained with the use of the pseudoinverse, are shown in Fig. 6, Fig. 7 and Fig. 8, while the normalized error reads

$$e_N = 0.0279. \tag{47}$$

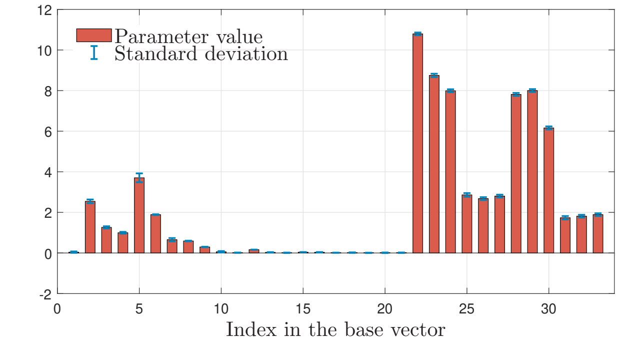

Fig. 5. Base parameters obtained by optimization

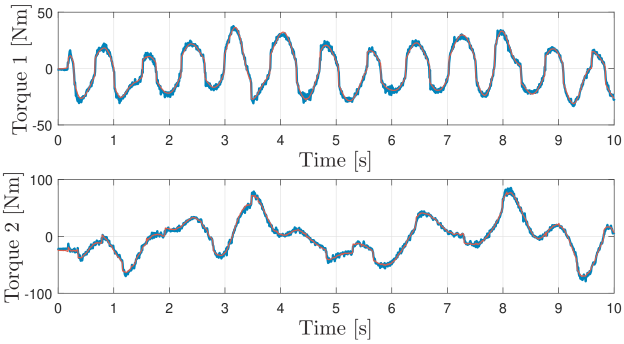

Fig. 6. Measured and predicted torques - pseudoinverse

For the parameter vector obtained using the optimization, predicted body torques are shown in Fig. 9, Fig. 10 and Fig. 11, while the normalized error for this vector is

$$e_N = 0.0301. \tag{48}$$

Next, calculated vectors of the base parameters are used for predicting torques obtained using the second excitation trajectory, shown in Fig. 12.

For the trajectory in Fig. 12, and using the base parameters obtained by pseudoinverse, prediction of torques are shown in Fig. 13, Fig. 14 and Fig. 15, while the normalized error is

$$e_N = 0.0152. \tag{49}$$

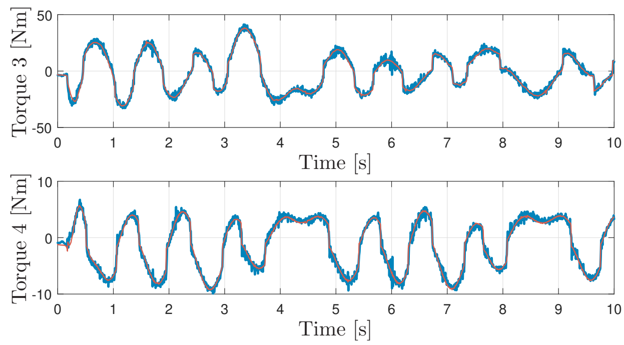

Fig. 7. Measured and predicted torques - pseudoinverse

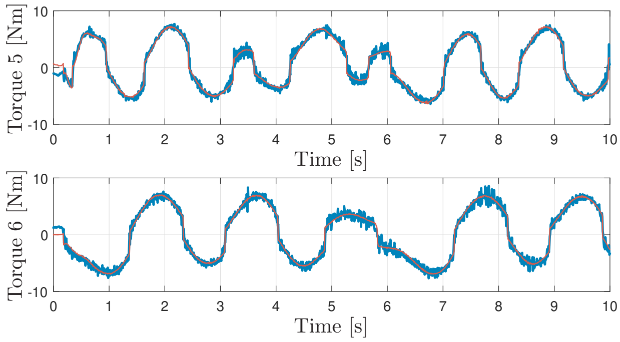

Fig. 8. Measured and predicted torques - pseudoinverse

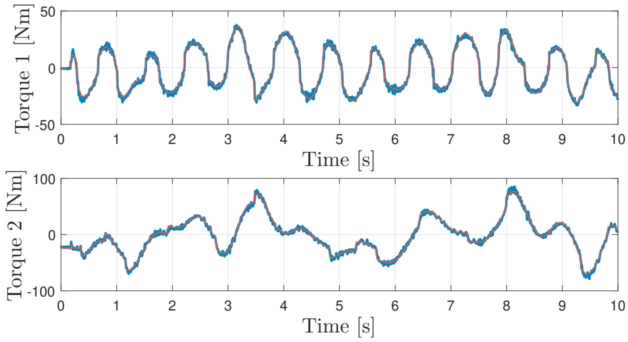

Fig. 9. Measured and predicted torques - optimization

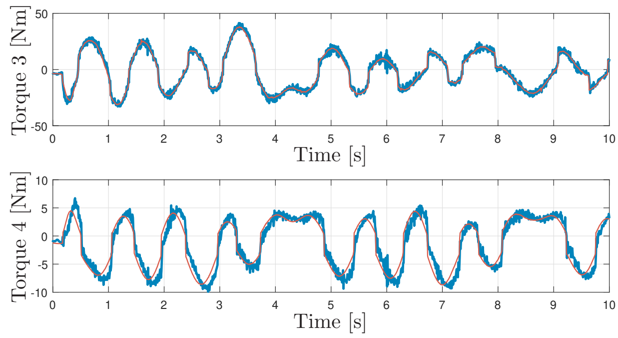

Fig. 10. Measured and predicted torques - optimization

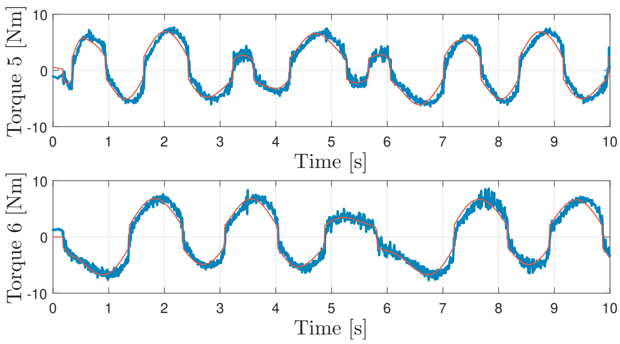

Fig. 11. Measured and predicted torques - optimization

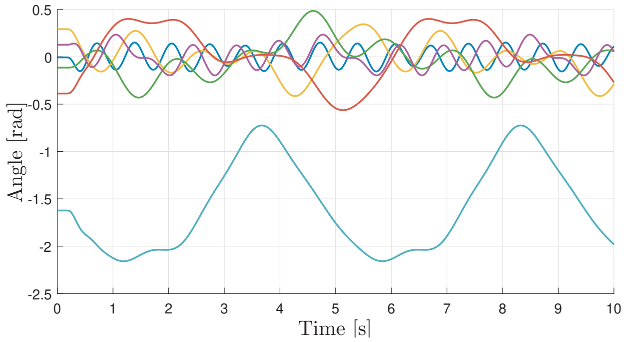

Fig. 12. Persistent excitation trajectories used for the parameter validation

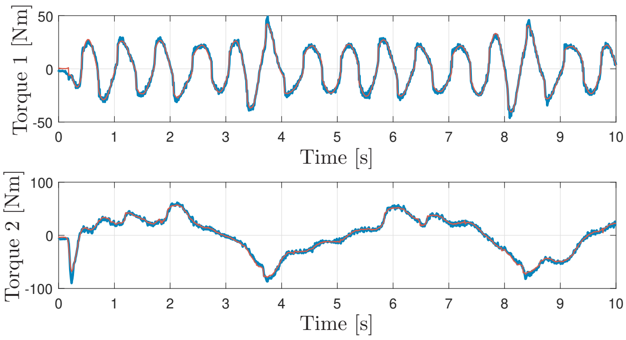

Fig. 13. Validation of the obtained base parameter vector, trajectory from Fig. 12 - pseudoinverse

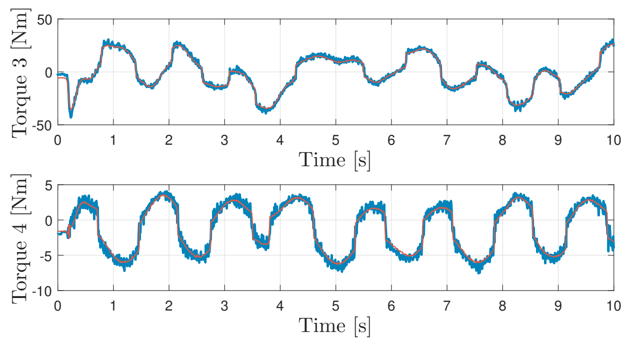

Fig. 14. Validation of the obtained base parameter vector, trajectory from Fig. 12 - pseudoinverse

Finally, torque predictions of the base parameters vector obtained by optimization, for trajectory in Fig. 12 are shown in Fig. 16, Fig. 17 and Fig. 18. The normalized error for this case is

$$e_N = 0.0163. \tag{50}$$

V. Conclusion

From the identification results several things can be seen. First, both base vectors can predict measured torques almost equally good. Although vector obtained by pseudoinverse has negative parameters corresponding to moment of inertia of the motor rotors, which is physically impossible, its torque

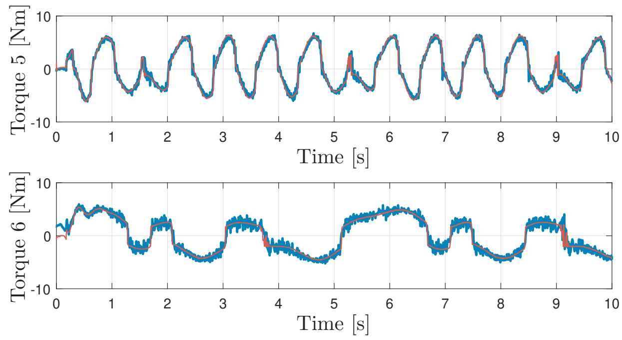

Fig. 15. Validation of the obtained base parameter vector, trajectory from Fig. 12 - pseudoinverse

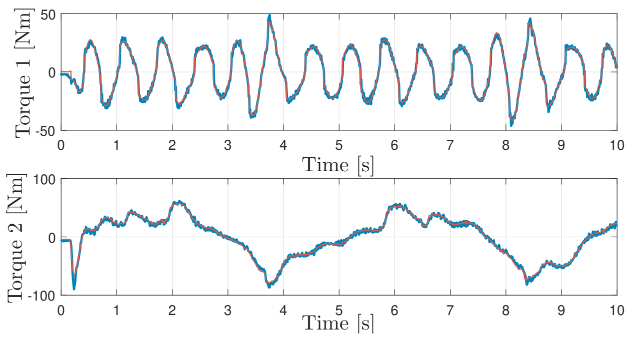

Fig. 16. Validation of the obtained base parameter vector, trajectory from Fig. 12 - optimization

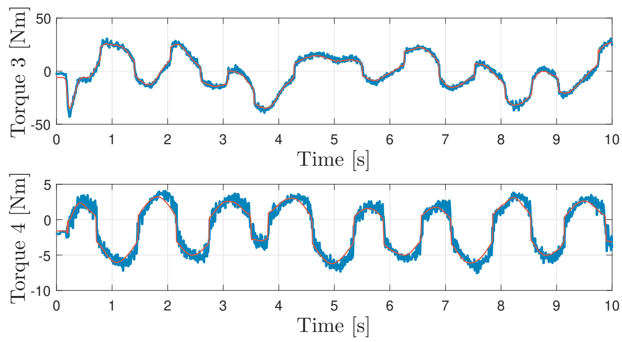

Fig. 17. Validation of the obtained base parameter vector, trajectory from Fig. 12 - optimization

predictions are a little bit better as can be seen from the corresponding normalized errors. However, the consequence of having physically impossible negative parameters is that the mass matrix is, for some robot configurations, not symmetric or negative definite and thus methods for mass matrix inversion tailored for symmetric positive definite matrices, like the Cholesky decomposition, can not be used.

At the end, note that on some figures showing torque predictions there is an error at zero time. This error is because of static friction which is greater that the here identified dynamic one.

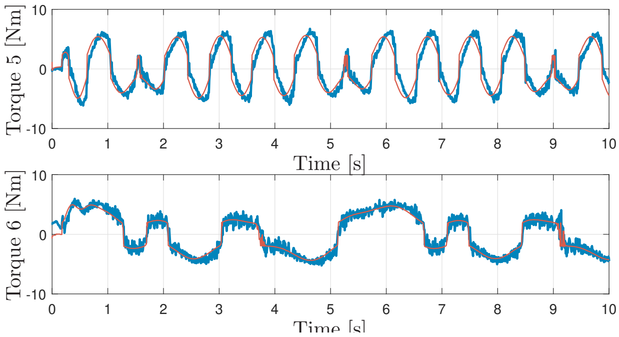

Fig. 18. Validation of the obtained base parameter vector, trajectory from Fig. 12 - optimization

ACKNOWLEDGMENT

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