A higher-order MRF based variational model for multiplicative noise reduction

Yunjin Chen, Wensen Feng, René Ranftl, Hong Qiao and Thomas Pock

Abstract—The Fields of Experts (FoE) image prior model, a filter-based higher-order Markov Random Fields (MRF) model, has been shown to be effective for many image restoration problems. Motivated by the successes of FoE-based approaches, in this letter we propose a novel variational model for multiplicative noise reduction based on the FoE image prior model. The resulting model corresponds to a non-convex minimization problem, which can be efficiently solved by a recently published non-convex optimization algorithm. Experimental results based on synthetic speckle noise and real synthetic aperture radar (SAR) images suggest that the performance of our proposed method is on par with the best published despeckling algorithm. Besides, our proposed model comes along with an additional advantage, that the inference is extremely efficient. Our GPU based implementation takes less than 1s to produce state-of-the-art despeckling performance.

Index Terms—speckle noise, despeckling, Fields of Experts, non-convex optimization, MRFs

I. INTRODUCTION

Images generated by coherent imaging modalities, e.g., synthetic aperture radar (SAR), ultrasound and laser imaging, inevitably come with multiplicative noise (also known as speckle), due to the coherent nature of the scattering phenomena. The presence of this noise prevents us from interpreting valuable information of images, such as textures, edges and point target, and therefore speckle reduction is often a necessary preprocessing step for successful use of classical image processing algorithms involving image segmentation and classification. The topic of speckle noise reduction (despeckling) has attracted a lot of research attention since early 1980s [17], [16]. At present it has been extensively studied. Roughly speaking, the major despeckling techniques fall into four categories: filtering based methods in (1) spatial domain; or (2) a transform domain, e.g., wavelet domain; (3) nonlocal filtering and (4) variational methods.

Early filtering techniques in the spatial domain are developed under the minimum mean square error (MMSE) criterion [17], and then progress to more sophisticated and promising maximum a posterior (MAP) approaches [16]. Recently, bilateral filtering has also been modified for despeckling [18]. The emergence of wavelet transform in the early of 1990s, opened the way to a new generation of despeckling techniques.

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There were intensive studies of wavelet based despeckling approaches, see, for instance [5], [3] and references therein.

The nonlocal approaches, which can take the advantage of self-similarity commonly present in natural as well as SAR images, have been already introduced to SAR despeckling [15], [20], [14]. By taking into account the peculiar features of multiplicative noise, the so-called SAR-BM3D algorithm [20], which is a SAR-oriented version of the well-known BM3D algorithm [12], exhibits the best published despeckling performance at present.

The last class of methods are variational ones, which minimize some appropriate energy functionals, consisting of a regularizer (also called image prior) and a data fitting term. Up to now, the well-known total variation (TV) has been widely used as a regularizer [22], [24], [4], and the total generalized variation (TGV) regularizer [6] also has been investigated in a recent work [14].

There is a long history of research on regularization technique for image processing problems. The recently proposed Fields of Experts (FoE) [21] image prior model (a higher-order filter-based MRF model), which explicitly characterizes the statistics properties of natural images, defines more effective variational models than hand-crafted regularizers, such as TV and TGV models. The variational model based on a discriminatively trained FoE prior can generate clear state-of-the-art performance for many image restoration problems, such as the additive white Gaussian noise (AWGN) denoising task, see for instance [7], [8].

Motivated by the results of [7], [8], it is interesting to investigate the FoE prior based model for despeckling. In this letter, we propose a novel variational approach for speckle removal, which involves an expressive image prior model - FoE and a highly efficient non-convex optimization algorithm - iPiano, recently proposed in [19]. Our proposed method obtains strongly competitive despeckling performance w.r.t. the state-of-the-art method - SAR-BM3D [20], meanwhile, preserve the property of computational efficiency.

II. PROPOSED VARIATIONAL MODELS FOR DESPECKLING

In this section, we propose FoE prior based variational models for speckle noise removal and in particular for SAR images. We introduce an efficient algorithm to solve the corresponding optimization problems.

Given an observation image $f$ corrupted by multiplicative noise, the FoE prior based despeckling model is defined as the following energy minimization problem

$$
\hat{u} = \arg \min_{u} E(u, f) = E_{\text{FoE}}(u) + D(u, f), \quad \text{(II.1)}
$$
There exists an alternative method to define a convex data term by using the classical Csizszár I-divergence model \[^{11}\]. Although the I-divergence data fitting term typically used in the context of Poisson noise, this seemingly inappropriate data term performs very well in the TV and TGV regularized variational models for despeckling \[^{22},\ 14\]. Therefore, we also study this variant for data term modeling in this work. Following previous works of modeling the SAR image intensity \[^{22},\ 14\], the I-divergence based data term for the amplitude model is given by

\[
D(u, f) = \frac{\lambda}{2}(u^2 - 2f^2\log u), \tag{II.5}
\]

which is strictly convex w.r.t. \(u > 0\). Then the variational model involving this convex data term is given by

\[
\arg\min_{u>0} \sum_{i=1}^{N_f} \theta_i \rho(k_i * u) + \frac{\lambda}{2} \langle u^2 - 2f^2\log u, 1 \rangle. \tag{II.6}
\]

C. Solving the variational despeckling models

Due to the non-convexity of the prior term, the proposed variational models impose generally very hard optimization problems, especially for the model \[^{11}\] with a non-convex data term. In this work, we resort to a recently published non-convex optimization algorithm - iPiano \[^{19}\] to solve them.

The iPiano algorithm is designed for a structured non-smooth non-convex optimization problem, which is composed of a smooth (possibly non-convex) function \(F\) and a convex (possibly non-smooth) function \(G\):

\[
\arg\min_{u} H(u) = F(u) + G(u). \tag{II.7}
\]

iPiano is an inertial force enhanced forward-backward splitting algorithm with the following basic update rule

\[
u_n+1 = (I + \alpha \partial G)^{-1}(u^n - \alpha \nabla F(u^n) + \beta(u^n - u^{n-1})),
\]

where \(\alpha\) and \(\beta\) are the step size parameters.

For the model \[^{11}\] with a non-convex data term, we can convert the data term to a convex function via commonly used logarithmic transformation (i.e., \(w = \log u\)). Therefore, the minimization problem is rewritten as

\[
\arg\min_{w} \sum_{i=1}^{N_f} \theta_i \rho(k_i * e^w) + \frac{\lambda}{2} \langle 2w + f^2e^{-2w}, 1 \rangle, \tag{II.8}
\]

with \(u = e^w\). Casting \[^{11}\] in the form of \[^{11}\], we see that \(F(w) = \sum_{i=1}^{N_f} \theta_i \rho(k_i * e^w)\) and \(G(w) = \frac{\lambda}{2} \langle 2w + f^2e^{-2w}, 1 \rangle\).

In order to use the iPiano algorithm, we need to calculate the gradient of \(F\) and the proximal map w.r.t. \(G\). It is easy to check that

\[
\nabla_w F = \sum_{i=1}^{N_f} \theta_i W K_i^T \rho'(K_i e^w),
\]

where \(K_i\) is an \(N \times N\) highly sparse matrix, implemented as 2D convolution of the image \(u\) with filter kernel \(k_i\), i.e., \(K_i u \leftrightarrow k_i * u, \ \rho'(K_i u) = (\rho'(K_i u_1), \ldots, \rho'(K_i u_N))^T \in \mathbb{R}^N\), with \(\rho'(x) = 2x/(1+x^2)\), and \(W = \text{diag}(e^w)\).

The proximal map w.r.t. \(G\) is given as the following minimization problem

\[
(I + \tau \partial G)^{-1}(\hat{w}) = \arg\min_{w} \frac{\|w - \hat{w}\|^2_2}{2} + \frac{\tau \lambda}{2} \langle 2w + f^2e^{-2w}, 1 \rangle. \tag{II.9}
\]
Instead of using a direct solver for (II.9) in terms of the Lambert W function [9], we utilized Newton’s method. We found that this scheme has a quite fast convergence (less than 10 iterations).

For the variational model (II.6), \( F(u) = \sum N_j \theta_i \rho(K_i * u) \), \( G(u) = \frac{1}{2} \left( u^2 - 2 f^\text{log}_u(1) \right) \). Then we have

\[
\nabla_u F = \sum_{i=1}^{N_j} \theta_i K_i^\tau \rho'(K_i u),
\]

and the proximal map w.r.t. \( G \) is given by the following pointwise calculation

\[
(I + \tau \partial G)^{-1}(\hat{u}) \Leftrightarrow u_p = \frac{\hat{u}_p + \sqrt{\hat{u}_p^2 + 4(1 + \tau \lambda)\tau \lambda f_p^2}}{2(1 + \tau \lambda)}
\]

III. EXPERIMENTAL RESULTS

In this section, we evaluated the performance of our proposed variational models based on images corrupted by synthetic speckle noise and real noisy SAR images. For synthetic experiments, we calculated the common measurements PSNR and SSIM index [23], and compared the results with the state-of-the-art algorithm - SAR-BM3D [20].

A. The influence of data term

We started with conducting a preliminary despeckling test on a commonly used natural image contaminated by multiplicative noise with \( L = 8 \), using our proposed variational models (II.8) and (II.6). We carefully tuned the parameter \( \lambda \) to insure that the corresponding variational models achieve the best performance. The despeckling results are shown in Figure 2(b) and (c).

A first impression from this result is: the variational model with an accurate data fitting term, which is exactly derived from the multiplicative noise model can lead to better result than the model with an “inappropriate” data term. But after having a closer look at the despeckling images, we found an interesting phenomena: these two methods possess complementary strengths and failure modes. For example, for the highly textured region (highlighted by the red rectangle), (II.8) works much better than (II.6); however, for the homogeneous region (highlighted by the white rectangle), (II.6) generates preferable result. Then an intuitive idea arises to integrate these two data terms so as to leverage their advantages.

As a result, a new variational model incorporating these two data terms comes out as follows

\[
\arg\min_e E_{\text{Foe}}(e^w) + \frac{\lambda_1}{2} (2 w + f^2 e^{-2w}) + \frac{\lambda_2}{2} (e^{2w} - 2 f^2 w),
\]

(III.1)

with \( u = e^w \). The data fitting term is still convex, and we can utilize iPiano to solve it. The proximal map w.r.t. \( G \) is also solved using Newton’s method.

We manually tuned the parameters \( \lambda_1 \) and \( \lambda_2 \), and conducted the same despeckling experiment. The final result is shown in Figure 2(d). One can see that the combined model indeed leads to significant improvement in terms of both PSNR and SSIM index value. From now on, we will use the combined model (III.1) for despeckling experiments.

B. Results on synthetic speckle noise

We first evaluated the performance of our proposed variational model (III.1) for despeckling based on synthetic noisy images. The results are compared to the best published despeckling algorithm - SAR-BM3D [20]. First of all, we considered three standard test images widely used in image processing community, see in Figure 3. The despeckling results at three representative numbers of look \( L = 1, 3, 8 \) are summarized in Table I. We made the following empirical choice for the parameters \( \lambda_1 \) and \( \lambda_2 \) for different \( L \): if \( L = 8 \), \( \lambda_1 = 550, \lambda_2 = 0.02 \); if \( L = 3 \), \( \lambda_1 = 310, \lambda_2 = 0.008 \); and if \( L = 1 \), \( \lambda_1 = 160, \lambda_2 = 0.004 \). A practical guideline to tune \( \lambda_1 \) and \( \lambda_2 \), as well as the implementation of the proposed approach can be found at our homepage [2].

From Table I one can see that these two competing approaches generate very similar results, with almost identical PSNR and SSIM values. We present three despeckling examples obtained by these two algorithms in Figure 4.

As we know, the despeckling performance of one particular method varies greatly for different image contents, in order to...
Table II

<table>
<thead>
<tr>
<th>L</th>
<th>PSNR (dB)</th>
<th>SSIM</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>17.42</td>
<td>0.3778</td>
</tr>
<tr>
<td>3</td>
<td>27.12</td>
<td>0.7756</td>
</tr>
<tr>
<td>1</td>
<td>24.85</td>
<td>0.6757</td>
</tr>
</tbody>
</table>

TABLE II

DE-speckling results on 68 Berkeley test images. The results are reported with average PSNR and SSIM values.

Fig. 4. Performance comparison to state-of-the-art algorithm - SAR-BM3D [20] for different L. The results are reported by PSNR/SSIM index.

(a) L = 1 (12.10/0.1595) (b) [20] (25.92/0.7432) (c) Ours (25.85/0.7771)

(d) L = 3 (16.48/0.2939) (e) [20] (27.48/0.8233) (f) Ours (28.38/0.8409)

(g) L = 8 (23.19/0.6981) (h) [20] (28.06/0.8663) (i) Ours (28.39/0.8647)

Fig. 5. Performance comparison of different algorithms on a real SAR image.

(a) Noisy image, L = 5  (b) SAR-BM3D [20]  (c) Ours

(d) PPBit [13] (e) TGV based [14]

Fig. 4. Performance comparison to state-of-the-art algorithm - SAR-BM3D [20] for different L. The results are reported by PSNR/SSIM index.

C. Results on a real SAR image

In order to demonstrate the effectiveness of our proposed method on real SAR image despeckling task, we conducted a despeckling test on a real noisy SAR image, which is taken by the radar system equipped with the JSTARS aircraft [11] (the number of looks L = 5). For this experiment, we set λ1 = 50, λ2 = 0.15 for our model.

The results of different algorithms are shown in Figure 5. One can see that the tiny structures in the image, especially the region highlighted by the white rectangle, are well-preserved by our approach after despeckling; however, they are smoothed out to different extents by other algorithms.

IV. CONCLUSION

In this letter, we have proposed a novel variational model for speckle noise reduction, based on an expressive image prior model - FoE model. Our new variational model poses a generally demanding non-convex optimization problem and we have used the recently proposed algorithm - iPiano to solve it efficiently. Numerical results on synthetic images corrupted by speckle noise and a real SAR image demonstrate that the performance of our method is clearly on par with the best published despeckling algorithm - SAR-BM3D. Furthermore, our model comes along with the additional advantage of simplicity and therefore well-suited to GPU programming. The GPU based implementation can conduct despeckling in less than 1s with state-of-the-art performance.
REFERENCES