EEG Independent Component Polarity Normalization by Convex Relaxation of a Two-Way Partitioning Problem

Nima Bigdely-Shamlo[1,2], Kenneth Kreutz-Delgado[1,2], and Scott Makeig[2]
[1]*Dept. of Electrical and Computer Engineering, UCSD, San Diego, USA;*
[2]*Swartz Center for Computational Neuroscience, UCSD, San Diego, USA*
Correspondence: N. Bigdely-Shamlo, UC San Diego, SCCN, 9500 Gilman Drive # 0559 La Jolla CA 92093-0559. E-mail: nima@sccn.ucsd.edu

Abstract. Independent Component Analysis (ICA) is a popular method for finding independent brain and artifact sources in EEG data. But there exists an inherent indeterminacy in the polarity of the scalp-map and activation associated with each Independent Component (IC) which can interfere with interpretation and use of the learned ICs from multiple subjects. In particular, these polarities are often required to be assigned in a coherent manner before IC activations can be used in multi-subject inference or in training Brain Computer Interface (BCI) algorithms that use data from more than one subject or recording session. Here we propose an IC polarity normalization method based on the convex relaxation of a two-way partitioning problem and evaulate its performance on a sample RSVP Target detection EEG dataset.

Keywords: EEG, BCI, ICA, Independent Component Analysis, Scalpmap, BCI, Polarity Normalization

1. Introduction

Independent Component Analysis (ICA) is a popular method for finding independent brain and artifact sources in EEG data [Bell and Sejnowski, 1995; Delorme and Makeig, 2003; Jung et al., 2001; Makeig et al., 1996]. Because ICA is a blind source separation problem there are two inherent indeterminacies, one is the order of ICs (irrelevant in EEG analysis) and the other is the scaling of activations and scalpmaps [Comon, 1994]. The latter is because the only observable quantity for each source is the potential recorded at scalp electrodes and it remains the same if both scalp-map and activation are multiplied by a non-zero scaling factor.

Normalization of scalp-maps so they all have an L-2 norm of one (in the appropriate physical units, e.g. microvolts) partially fixes this problem but the ambiguity in scalp-map polarities still remains. To perform multi-subject analyses and online classifier learning, it is desirable to normalize the IC scalp-map polarities before calculating averages over IC activities associated with experiment events (otherwise, incompatible polarities may results in signal cancellation). Polarity normalization could be achieved by making ICs with similar scalp-maps to have similar polarities (which corresponds to the inner product of their scalp-map vectors to being positive, when signals are real). We show how this can be achieved using convex relaxation of a two-way partitioning problem.

2. Material and Methods

We want to find a vector of scalp-map polarities $x \in R^n, x_i \in \{-1, 1\}$ which minimizes the negative sum of normalized scalp-map inner products , $W_{i,j} = \frac{-s_i s_j^T}{\|s_i\|\|s_j\|}$ with S_i as the i^th^ column of scalp projection matrix S. The inner products comprise the components of a matrix W. The scalar quantity $x^T W x$, subject to the constraint that the components of x are either +1 or -1, provides an aggregate measure of the total dissimilarity across scalp-maps which is the quantity that we desire to minimize with respect to the x-component signs. This is because $x^T W x = \sum_{i,j=1..n} x_i x_j W_{i,j}$ gives the sum of scalp-map inner products after changing their polarities according to the signs of the components of the x vector. We can formulate this as the following optimization problem [Boyd and Vandenberghe, 2004]:

$$\text{minimize } f_0 = x^T W x, \text{ subject to } \quad x_i \in \{-1, 1\} \tag{1}$$

Note that this problem is not convex since the domain of $x_i \in \{-1, 1\}$ is not convex, re-writing this equation in the equivalent form

$$\text{minimize tr}(WX), \text{ subject to } X \geq 0, \text{ rank}(X) = 1, X_{ii} = 1, i = 1, \dots, n \tag{2}$$

with variable $X \in S^n, X = xx^t$ (since $\text{tr}(WX) = \sum_{i,j=1..n} X_{i,j} W_{i,j}$ and $X_{ij} = x_i x_j$). We now drop the $\text{rank}(X) = 1$ constraint to obtain the convex optimization problem. Since this new problem is less constrained, its optimal value will provide a lower bound on the optimal value of the original problem (3). After solving problem (4), we are able to obtain the vector of polarities x from $X = x^t x$ using the real Schur decomposition $X = UTU^T$ where U is a unitary matrix ($U^{-1} = U^{T}$) and T is an upper triangular (in this symmetric case, diagonal) matrix. If X is low rank (recall that we have relaxed the problem by allowing it to have a rank more than one), we can ignore all elements of this matrix, except the highest value on the diagonal, located at $T_{n,n}$.

3. Results

We used 266 scalp-maps from ICA decompositions of 15 sessions associated with 7 subjects during a Rapid Serial Visual Presentation experiment [Bigdely-Shamlo et al., 2008]. Fig. 1 (left) shows a number of these scalp-maps with solid rectangles enclosing sample IC pairs with a similar pattern but reversed polarities. Our goal is to minimize such occurrences by changing the polarity of some of these scalp-maps. Fig. 1 (right) shows scalp maps of the ICs from Fig. 1 (left) with normalized polarities using the covex relaxation method described above. The polarities of all the pairs displayed here are corrected after applying the normalization method.

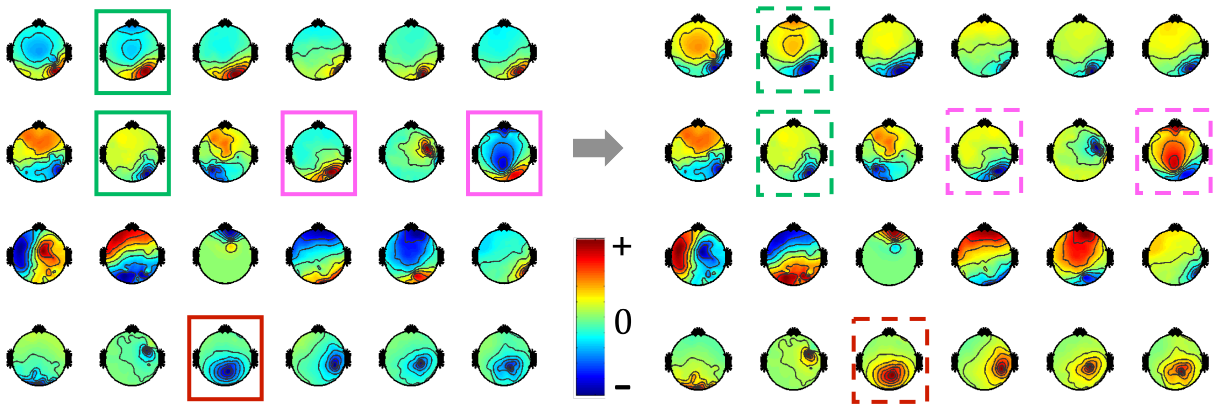

Figure 1. *(left) Sample scalp-maps before polarity normalization. IC pairs for which one IC is a candidates for polarity reversal are surrounded by colored rectangles (right) same scalp-maps after polarity normalization using convex relaxation method. Dashed rectangles indicate IC pairs highlighted on the left as candidate for polarity change. The polarities of all the pairs displayed here are corrected after applying convex polarity normalization.*

4. Discussion

We showed how scalp map polarities may be normalized using a convex relaxation of the two-way partitioning problem. This method could improve applications of ICA-based multi-subject BCI and EEG analysis using temporal features by normalizing the polarity of these features across components from different subjects and sessions.

References

Bell AJ, Sejnowski TJ. An Information Maximization Approach to Blind Separation and Blind Deconvolution. *Neural Comput*, 7(6):1129-1159, 1995.

Bigdely-Shamlo N, Vankov A, Ramirez RR, Makeig S. Brain Activity-Based Image Classification From Rapid Serial Visual Presentation. *IEEE Trans Neural Syst Rehabil Eng*, 16(5):432-441, 2008.

Boyd SP, Vandenberghe L. Convex optimization. Cambridge, UK; New York: Cambridge University Press. xiii, 716pp, 2004.

Comon P. Independent component analysis, A new concept? *Signal Proc*, 36(3):287-314, 2994.

Delorme A, Makeig S. EEG changes accompanying learned regulation of 12-Hz EEG activity. *IEEE Trans Neural Syst Rehabil Eng*, 11(2):133-137, 2003.

Jung TP, Makeig S, McKeown MJ, Bell AJ, Lee TW, Sejnowski TJ. Imaging brain dynamics using independent component analysis. *Proc IEEE*, 89(7):1107-1122, 2001.

Makeig S, Bell AJ, Jung TP, Sejnowski T. Independent component analysis of electroencephalographic data. *Advances in Neural Information Processing Systems*, 8:145-151, 1996.