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Energy Optimal Control of an Industrial Robot by using the Adjoint Method

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Abstract
The main goal of this contribution is to determine the excitation of an industrial robot, such that the energy consumption becomes a minimum during the manipulation of the tool center point (TCP) from a start position to a given end point within a predefined time. Such tasks can be restated as optimization problems where the functional to be minimized consists of the endpoint error and a measure for the energy. The gradient of this functional can be calculated by solving a linear differential equation, called the adjoint system. On the one hand the minimum of the cost functional can be achieved by the method of steepest descent where a proper step size has to be found or on the other hand by a Quasi-Newton algorithm where the Hessian can be appreciated. The theory is applied to a six-axis robot and the identification leads to a reduction of 47% of the signal energy.

Keywords: *optimal control, multibody dynamics, adjoint system, optimization, calculus of variation.*

1. Introduction

In this contribution an approach to such inverse dynamical problems is presented. It starts from an optimal control formulation of the problem by introducing a cost functional which has to be minimized subject to a system of differential equations (c.f. [1, 2]). The gradient computation of the cost functional is based on the so called adjoint method. Due to better convergence a Quasi-Newton method is used instead of the simple gradient method. Therefore, the Hessian matrix is approximated by using the BFGS-algorithm.

The adjoint method is already used in a wide range of optimization problems in engineering sciences. Especially, in the field of multibody systems, the computation of the gradient of the cost function is often the bottleneck for computational efficiency and the adjoint method serves as the most efficient strategy in this case. The basic idea of the adjoint method is the introduction of additional *adjoint* variables determined by a set of adjoint differential equations from which the gradient can be computed straightforward. This main idea directly corresponds to the gradient technique for trajectory optimization pioneered by Bryson and Ho [3].

Various authors have utilized the adjoint method in the sensitivity analysis of multibody system, as e.g., [4, 5]. Bottasso et al. [6] presented a combined indirect approach of the adjoint method in multibody dynamics for solving inverse dynamics and trajectory optimization problems, also similar to the ideas presented in [7].

For a signal energy optimal manipulation of the robot a cost functional is introduced, which consists of the quadratic input signals in every time step and of a so-called Scrap-function which defines the end point deviation.

The identified movements were tested on a PUMA six axis robot. With the measured control variables the required energy was evaluated. Based on this test data a considerably energy reduction was detected.

2. Problem definition

At first, let us consider a nonlinear dynamical system

$$\begin{aligned}\dot{\boldsymbol{q}} &= \boldsymbol{v}\\ \boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{v}} &= \tilde{\boldsymbol{f}}(\boldsymbol{q},\boldsymbol{v},\boldsymbol{u},t),\end{aligned} \tag{1}$$

where $\boldsymbol{q} \in \mathbb{R}^n$ is the vector of generalized coordinates and $\boldsymbol{v} \in \mathbb{R}^n$ is the vector of generalized velocities. In addition, $\boldsymbol{M}$ is the $n \times n$ mass matrix and $\tilde{\boldsymbol{f}} \in \mathbb{R}^n$ the force vector. The vector $\boldsymbol{u}$ indicates the control variables in an opened or enclosed region $\Gamma \subseteq \mathbb{R}^m$. By introducing the vector of state variables $\boldsymbol{x}^\mathsf{T} = (\boldsymbol{q}^\mathsf{T}\ \boldsymbol{v}^\mathsf{T})$ we may rewrite Equation (1) by

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x},\boldsymbol{u},t) \qquad \boldsymbol{x}(t_0) = \boldsymbol{x}_0. \tag{2}$$

In general the force vector $\boldsymbol{f}$ is a continuous vector field which depends on the states $\boldsymbol{x}$, controls $\boldsymbol{u}$ and on time t. In robotics, the position and velocity of the tool center point (TCP) will be of particular interest instead of the joint angles and angular velocities. Hence, the system output $\boldsymbol{y} \in \mathbb{R}^l$ is given by

$$\boldsymbol{y} = \boldsymbol{g}(\boldsymbol{x}).$$

In order to meet a predefined end point we have to satisfy the boundary condition

$$\boldsymbol{g}(\boldsymbol{x}(t_f)) = \bar{\boldsymbol{y}}. \tag{3}$$

However, we substitute the boundary condition of Equation (3) by the optimal control problem

$$\begin{aligned}\dot{\boldsymbol{x}} &= \boldsymbol{f}(\boldsymbol{x},\boldsymbol{u},t)\\ J &= \int_{t_0}^{t_f} h(\boldsymbol{x},\boldsymbol{u},t)\ \mathrm{d}t + S(t_f,\boldsymbol{x}(t_f)) \longrightarrow \text{Min.}\end{aligned} \tag{4}$$

where the integral describes the energy consumption and the *Scrap-function* S includes the end point error. If the closed region Γ is not empty the solution of the *optimal control* problem of Equation (4) leads to an energy optimal manipulation of the dynamical system of Equation (2).

3. Gradient computation

To determine the gradient of the cost functional (4) we first add zero terms to it:

$$J = \int_{t_0}^{t_f} h(\boldsymbol{x},\boldsymbol{u},t) + \boldsymbol{p}^\mathsf{T} \underbrace{[\boldsymbol{f}(\boldsymbol{x},\boldsymbol{u},t) - \dot{\boldsymbol{x}}]}_{=0 \text{ Eq. (2)}} \mathrm{d}t + S(t_f,\boldsymbol{x}(t_f)) \tag{5}$$

The Lagrange-multipliers $\boldsymbol{p}$ are denoted as adjoint variables and are arbitrary at this point. Integration by parts of the term $\int \boldsymbol{p}\dot{\boldsymbol{x}}\,\mathrm{d}t$ leads to

$$J = \int_{t_0}^{t_f} \left(H + \dot{\boldsymbol{p}}^\mathsf{T}\boldsymbol{x}\right)\mathrm{d}t + S(t_f, \boldsymbol{x}(t_f)) - \boldsymbol{p}^\mathsf{T}\boldsymbol{x}\Big|_{t_0}^{t_f}, \tag{6}$$

where the *Hamiltonian* $H(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{p}, t) = h(\boldsymbol{x}, \boldsymbol{u}, t) + \boldsymbol{p}^\mathsf{T}\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, t)$ is introduced. In order to find a minimum of the cost functional J with respect to $\boldsymbol{u}$ we consider the variation of J according to a small change $\delta\boldsymbol{u}$ which is given by

$$\delta J = \int_{t_0}^{t_f} \left[\left(H_{\boldsymbol{x}} + \dot{\boldsymbol{p}}^\mathsf{T}\right)\delta\boldsymbol{x} + H_{\boldsymbol{u}}\delta\boldsymbol{u}\right]\mathrm{d}t + \left[S_{\boldsymbol{x}}(t_f, \boldsymbol{x}(t_f)) - \boldsymbol{p}^\mathsf{T}(t_f)\right]\delta\boldsymbol{x}(t_f) + \boldsymbol{p}^\mathsf{T}(t_0)\delta\boldsymbol{x}(t_0). \tag{7}$$

Due to the fact that no variation of the states at $t = t_0$ is allowed, the term $\boldsymbol{p}(t_0)\delta\boldsymbol{x}(t_0)$ is zero. If the adjoint variables are defined, such that

$$\dot{\boldsymbol{p}}^\mathsf{T} = -H_{\boldsymbol{x}} \quad \text{and} \quad \boldsymbol{p}^\mathsf{T}(t_f) = S_{\boldsymbol{x}}(t_f, \boldsymbol{x}(t_f)), \tag{8}$$

the complex relations between $\delta\boldsymbol{x}$ and $\delta\boldsymbol{u}$ need not to be computed and the variation of J according to Equation (7) is reduced to

$$\delta J = \int_{t_0}^{t_f} H_{\boldsymbol{u}}\delta\boldsymbol{u}\,\mathrm{d}t. \tag{9}$$

Equation (8) is a linear and time-variant system of differential equations which have to be solved backwards in time starting at $t = t_f$. Hence, the largest possible increase of δJ is obtained, if $\delta\boldsymbol{u}(t)$ is chosen in the direction of $H_{\boldsymbol{u}}^\mathsf{T}$. For that reason $H_{\boldsymbol{u}}^\mathsf{T}$ may be considered as the gradient of the cost functional $J(\boldsymbol{u})$.

4. Numerical determination of the optimal control

Based on the adjoint gradient computation, outlined in the previous section, we may now search for a control $\boldsymbol{u}$ which minimizes the objective functional J. First of all, the method of steepest descent is described, where we always walk a certain distance along the negative gradient until we end up in a local minimum of J. Due to the costly line search step during every iteration and the slow convergence the gradient method is extended to a Quasi-Newton method. Therefore, we have to solve the problem of finding $\boldsymbol{u}$ such that the gradient becomes zero.

4.1. The Method of Steepest Descent

The method of steepest descent tries to find a minimum of a function or, subsequently, of a functional by walking always along the direction of its negative gradient. This concept has first been developed to optimal control problems by H.J. Kelley [8] and A.E. Bryson [9].

The gradient is already derived from the adjoint system which is shown in Section 3. Now we use $H_{\boldsymbol{u}}^\mathsf{T}$ and simply walk a short distance along the negative gradient of J. By reason of numerics the continuous functions are discretized. Hence, the cost functional reads

$$J(\boldsymbol{u}) \approx \hat{J}(\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_N) \tag{10}$$

where $\boldsymbol{u}_i = \boldsymbol{u}(t_i)$ and $t_1, \ldots, t_N$ is a sequence of consecutive time steps in the interval $[t_0, t_f]$. A variation of the controls $\boldsymbol{u}_i$ leads to a variation of the cost functional

$$\delta \hat{J} = \sum_{i=1}^{N} \frac{\partial \hat{J}}{\partial \boldsymbol{u}_i} \delta \boldsymbol{u}_i.$$

On the other hand, the variation $\delta \hat{J}$ can be expressed by Equation (9) which, after discretisation, results in

$$\delta \hat{J} = \sum_{i=1}^{N} H_{\boldsymbol{u},i} \Delta t_i \delta \boldsymbol{u}_i$$

where $H_{\boldsymbol{u},i}$ is the evaluation of $H_{\boldsymbol{u}}$ at $t = t_i$. Hence, the gradient of the discretised functional may be identified as

$$\frac{\partial \hat{J}}{\partial \boldsymbol{u}_i} = H_{\boldsymbol{u},i} \Delta t_i$$

in which $\Delta t_i = t_i - t_{i-1}$. For walking in the direction of the negative gradient a small number $\kappa > 0$ has to be chosen to get the increment

$$\delta \boldsymbol{u}_i = -\kappa H_{\boldsymbol{u},i}^{\mathsf{T}} \Delta t_i. \tag{11}$$

If κ is sufficiently small, the updated control $\boldsymbol{u}_i + \delta \boldsymbol{u}_i$ will always reduce the cost functional J. However, finding the number κ such that J is reduced may require several simulations of the system equations. For that purpose, the increments given by Equation (11) are considered as functions of κ. After solving the equations of motion with $\boldsymbol{u} + \delta \boldsymbol{u}$ as inputs also the objective function J becomes, ultimately, a function of κ. By means of a line search algorithm one may find a number κ in a predefined interval $[0, \kappa_{\text{max}}]$ which minimizes J.

4.2. Application of a Quasi-Newton Method

It is well known that the convergence of the gradient method is rather slow, especially near the optimal solution. Hence, a Newton method provides an alternative approach to find the minimum of the cost functional J. The basic idea is the following one: If $\hat{\boldsymbol{u}} = (\boldsymbol{u}_1^{\mathsf{T}}, \boldsymbol{u}_2^{\mathsf{T}}, \ldots, \boldsymbol{u}_N^{\mathsf{T}})^{\mathsf{T}}$ is defined by a zero gradient, i.e. by the equations

$$\nabla \hat{J} = \left[\frac{\partial \hat{J}}{\partial \boldsymbol{u}_1}, \cdots, \frac{\partial \hat{J}}{\partial \boldsymbol{u}_N} \right]^{\mathsf{T}} = \mathbf{0}$$

which can be solved for $\hat{\boldsymbol{u}}$ by Newton's method. However, the Hessian $\boldsymbol{H}$ is required for that purpose. To avoid the full computation of $\boldsymbol{H}$, which would be extremely time consuming, several quasi-Newton methods have been developed. They all approximate the Hessian by using the gradients of successive Newton-iterations. For example, the Hessian can be estimated efficiently by the well known *Broyden-Fletcher-Goldfarb-Shanno* (BFGS)-Algorithm (c.f. [10]). Even its inverse can be efficiently obtained by applying the *Sherman-Morrison formula* (c.f. [11]).

We compute an approximation $\tilde{\boldsymbol{H}}^{-1}$ of the inverse of the Hessian from the BFGS-algorithm. Then, an increment $\delta \hat{\boldsymbol{u}}$ of the discretized control signal is given by

$$\begin{pmatrix} \delta \boldsymbol{u}_1 \\ \delta \boldsymbol{u}_2 \\ \vdots \\ \delta \boldsymbol{u}_N \end{pmatrix} = -\tilde{\boldsymbol{H}}^{-1} \nabla \hat{J} \tag{12}$$

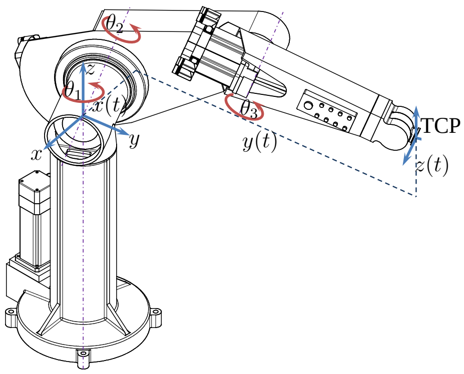

Figure 1. Schematics of the six-axis PUMA robot

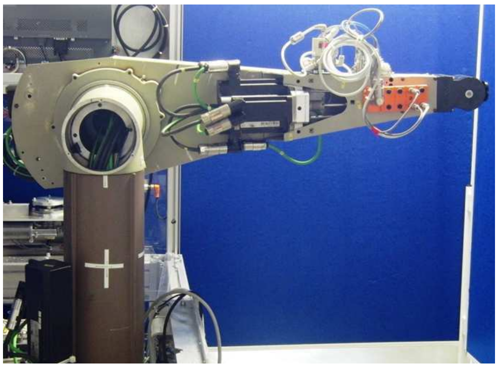

Figure 2. Image of the six-axis PUMA robot

Note, that it is strongly recommended to use a quasi-Newton method which directly approximates the inverse of the Hessian. Otherwise, if the original Hessian is computed, a very large and dense matrix must be inverted, since the number of components of J might become large.

The inverse of the Hessian after $k+1$ iterations is given by

$$\tilde{\boldsymbol{H}}_{k+1}^{-1} = \left(\boldsymbol{I} - \frac{\boldsymbol{p}_k \boldsymbol{q}_k^\mathsf{T}}{\boldsymbol{q}_k^\mathsf{T} \boldsymbol{p}_k}\right) \tilde{\boldsymbol{H}}_k^{-1} \left(\boldsymbol{I} - \frac{\boldsymbol{q}_k \boldsymbol{p}_k^\mathsf{T}}{\boldsymbol{q}_k^\mathsf{T} \boldsymbol{p}_k}\right) + \frac{\boldsymbol{p}_k \boldsymbol{p}_k^\mathsf{T}}{\boldsymbol{q}_k^\mathsf{T} \boldsymbol{p}_k} \tag{13}$$

where $\boldsymbol{I}$ is the identity matrix, $\boldsymbol{p}_k$ is the gradient direction of the k^{th}-iteration and $\boldsymbol{q}_k$ is the change of the gradient during the last iteration.

5. Application to the six-axis-robot

The presented method is used to minimize the signal energy consumption of the robot which is depicted in Figure 1. The reason why we have chosen this robot is that a lot of different parameters are available which are necessary for the evaluation and verification of the results. Afterwards, the simulation results are verified at a real six-axis-robot which is shown in Figure 2.

5.1. Problem definition

The system consists of three degrees of freedom, θ_1, θ_2 and θ_3 which denote the relative rotation angles of the joints. Due to the complicated structure of the equations of motion and the minor influence on the energy consumption the three wrist joints are fixed. First of all the equations of motion are derived and have the form $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, t)$ with the initial condition $\boldsymbol{x}(t_0) = \boldsymbol{x}_0$ and where $\boldsymbol{u} = [M_1, M_2, M_3]^\mathsf{T}$ contains the torques of the motors and $\boldsymbol{x} = [\theta_1, \theta_2, \theta_3, \dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3]^\mathsf{T}$ is the vector of states of the dynamical system. The system output $\boldsymbol{y} = \boldsymbol{g}(\boldsymbol{x})$ is a nonlinear function which depends on the states and describes the coordinates of the tool center point $\boldsymbol{y} = [x(t), y(t), z(t)]^\mathsf{T}$.

For the energy optimal manipulation of the robot from a start-point $\boldsymbol{x}_0$ to a given end-point $\overline{\boldsymbol{y}}$, $\dot{\overline{\boldsymbol{y}}}$ (c.f. Table 1) within a predefined time t_f we define the cost functional in the form

$$J = \underbrace{\int_{t_0}^{t_f} \boldsymbol{u}^\mathsf{T} \boldsymbol{u} \, \mathrm{d}t}_{\text{signal-energy}} + S(t_f, \boldsymbol{x}(t_f)). \tag{14}$$

Table 1. Start and end position of the robot

	start position	final position	start velocity	final velocity
θ_1	$0\,°$	$-90\,°$	$0\,\mathrm{rad/s}$	$0\,\mathrm{rad/s}$
θ_2	$0\,°$	$-10\,°$	$0\,\mathrm{rad/s}$	$0\,\mathrm{rad/s}$
θ_3	$0\,°$	$45\,°$	$0\,\mathrm{rad/s}$	$0\,\mathrm{rad/s}$
x_{TCP}	$-0.15320\,\mathrm{m}$	$0.81441\,\mathrm{m}$	$0\,\mathrm{m/s}$	$0\,\mathrm{m/s}$
y_{TCP}	$0.92112\,\mathrm{m}$	$-0.15320\,\mathrm{m}$	$0\,\mathrm{m/s}$	$0\,\mathrm{m/s}$
z_{TCP}	$0.02032\,\mathrm{m}$	$0.22233\,\mathrm{m}$	$0\,\mathrm{m/s}$	$0\,\mathrm{m/s}$

which contains the quadratical signal energy to be minimized. The scrap-function S of Equation (14) describes the endpoint error and is specified by

$$S(\boldsymbol{x}, t) = \alpha \left\{ \beta \left[\boldsymbol{y}(\boldsymbol{x}) - \overline{\boldsymbol{y}} \right]^2 + \left[\frac{\partial \boldsymbol{y}}{\partial \boldsymbol{q}} \dot{\boldsymbol{q}} - \dot{\overline{\boldsymbol{y}}} \right]^2 \right\} \tag{15}$$

where α and β are proper weighting factors and $\overline{\boldsymbol{y}}, \dot{\overline{\boldsymbol{y}}}$ contains the position and velocity of the endpoint in coordinates of the system output.

5.2. Results

The identification process of the signal energy optimal manipulation was started with the standard motion which is given from the robot controller. The results were verified on a real six-axis robot at the home institution. Hence, the data of the experiment and the simulation results are summarized in Figure 3. On the vertical axis the signal energy consumption is plotted over the time. It can be seen, that the standard manipulation wastes a lot of energy at the beginning and at the end of the motion due to the abrupt acceleration of the bodies. However, the signal energy optimal manipulation starts with a smooth movement of the heavy bodies. Therefore, the maximal speed of the axis have to be higher in comparison to the standard manipulation to reach the endpoint in the same period of time. As a result the reduction of the signal energy after the optimization process is about 47% with respect

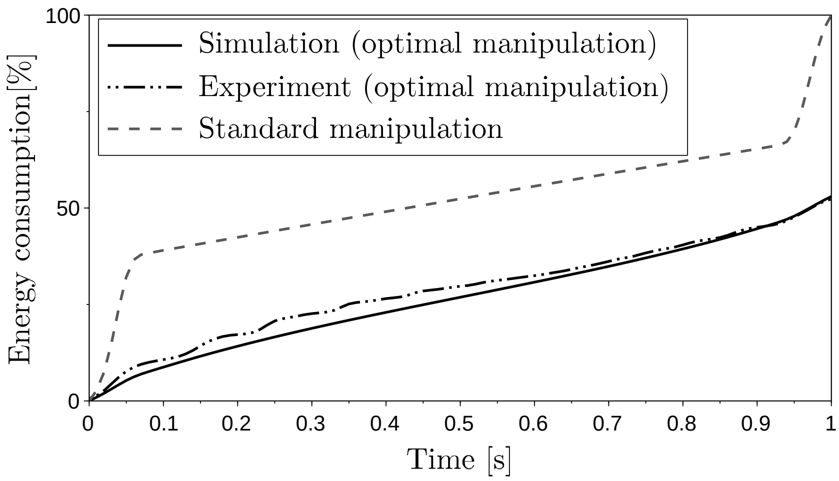

Figure 3. Build-up of the mechanical energy consumption

to the standard manipulation of the robot control.

In the upper part of Figure 4 the joint angles of the signal energy optimal manipulation in comparison to the standard manipulation of the robot are plotted over time. Obviously, the smooth characteristic of the optimal solution, which corresponds to the dashed line can be seen. However, the standard manipulation, which corresponds to the solid line, shows the commonly used standard motion calculated by the robot controller. In the lower part of Figure 4 the torques are depicted over the time. Here, the smooth characteristic of the optimized solution can be seen clearly.

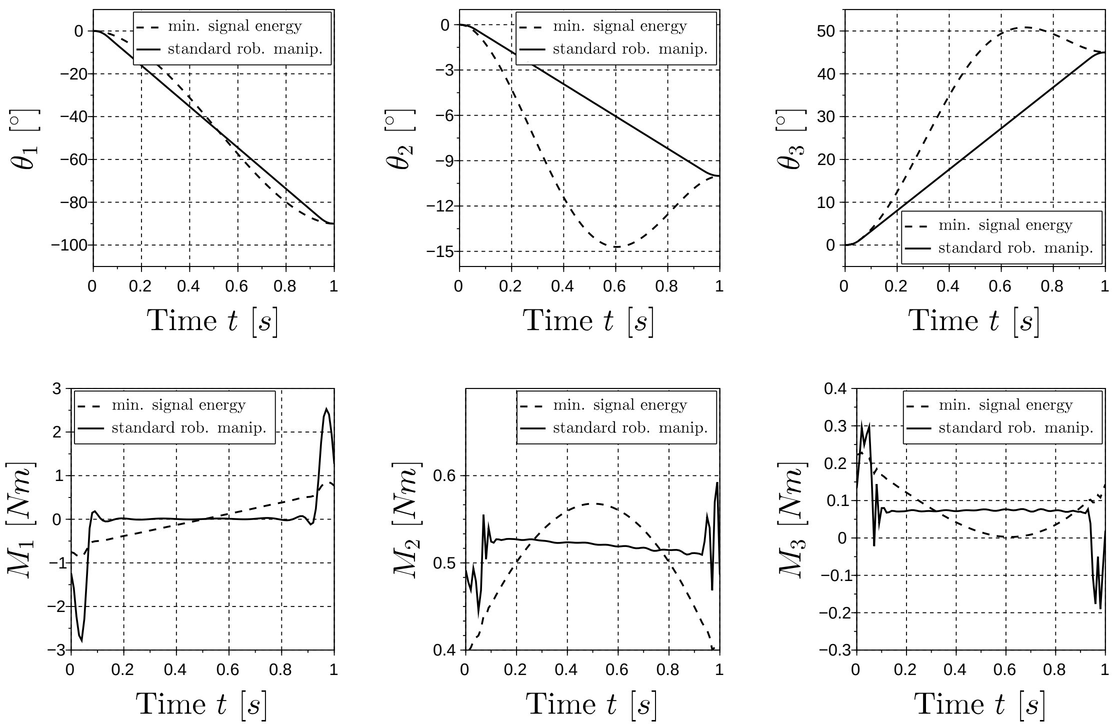

Figure 4. Trajectory of the states and torques in the axis

6. Conclusions and outlook

To reach a desired endpoint within a predefined time, the definition of a Scrap-function is required only. In addition, various requests to the system behavior can be considered in the integral part of the cost functional, such as the signal energy of an industrial robot.

This paper should reveal that the trajectory with minimal signal energy does not lead automatically to the mechanical energy optimal manipulation of the robot. Nevertheless, in practice such quadratic input terms are often used because this leads to less stress of the components. In simply terms you can say that the electrical parts are protected against overheating and the operation life span is increased additionally if the torques remain small and smooth over the manipulations.

For the results in Section 5.2. we neglected the three degrees of freedom of the wrist and fixed them to keep the equations of motion and the necessary matrices simple. However, if we consider this joint angles in the system equations it is possible to reach a predefined endpoint in different ways. This means that more than one final configuration of the robot exists which meet the end point in the coordinates of the tool center point.

Furthermore, the proposed identification can be done during operation. Instead of the forward simulation the measures of the previous manipulation can be used to solve the adjoint system and calculate the gradient. Hence, the defined cost functional, and therefore the signal energy, decreases during the manipulation of the robot. A big advantage is that it is not necessary to exchange any part of the robot, only an update of the robot control is required.

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