

# A NEW CLASS OF ROBUST RF PULSES BY OPTIMAL CONTROL

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**Abstract**— This work aimed to design and investigate inversion pulses that are robust among  $B_0$  and  $B_1$  inhomogeneities with a minimized pulse duration by optimal control. The optimized RF pulse was compared numerically to a state-of-the-art adiabatic RF pulse and a customized adiabatic one. All three RF pulses were investigated in extensive measurements on a 3T MRI system. Phantom measurements were performed to examine robustness with respect to  $B_0$  and  $B_1$ . In vivo measurements of the knee emphasized the practicability of the proposed RF pulse which is shown to be robust among variations within  $B_0$  and  $B_1$ .

**Keywords**— RF pulses, optimal control, robustness,  $B_0$  inhomogeneities,  $B_1$  inhomogeneities

## Introduction

Different MRI application experiments require inversion radio frequency pulses, i.e. pulses with a flip angle of  $180^\circ$ . However, for many applications, inhomogeneities within the  $B_0$  and  $B_1$  field are an issue, [1-9]. For special applications such as arterial spin labeling, even small deviations from the required exact inversion are unsatisfactory, [10]. Strong improvements regarding inversion efficiency could be achieved by using composite [6] or adiabatic [3] RF pulses, but at the cost of higher pulse energy and prolonged pulse duration. The design of RF pulses by optimal control has shown in the past that even conflicting requirements such as best pulse performance, short pulse duration, and limited pulse energy can be combined and fulfilled by using the entire parameter space [11-15]. This approach was already extended to account for  $B_0$  and  $B_1$  inhomogeneities, [4,16,17]. In [18], an ensemble-based optimal control formalism was used to include a time-minimal formalism and optimize for  $B_0$  and  $B_1$  robust inversion pulses. This work aims to compare the optimized RF pulse to state-of-the-art adiabatic RF pulses in phantom and in vivo measurements on a 3T MR system.

## Theory and Methods

The goal of the optimization is to design RF pulses with robustness over a wide range of  $B_0$  and  $B_1$  variations, [18]. Therefore, we include  $B_1$  scalings of 70% to 130% (i.e. a scale of the amplitude of the RF pulse

by those factors) and  $B_0$  offsets of  $\pm 5$ ppm at 3T into the optimization. The optimization itself uses exact discrete derivatives supplied by adjoint calculus within a trust-region, semi-smooth quasi-Newton framework [13]. We use a 10ms RF pulse with random magnitude and random phase as initial. During optimization, the relaxation times were chosen to coincide with those of our cylindrical MR phantom (plastic bottle with diameter 14cm, length 42,5cm, filled with  $H_2O$  and contrast agent resulting in  $T_1=102$ ms,  $T_2=81$ ms at 3T). The underlying Bloch equations were solved using a symmetric operator splitting allowing for the inclusion of the relaxation effects, [19].

To compare the optimized pulse, two adiabatic, hyperbolic secant pulses are introduced. The first one (**HS1**) is commonly implemented for arterial spin labeling applications [10] and has a long pulse duration of 15.36ms. The second one (**HS2**) was designed so that the pulse duration and bandwidth coincide with those of the optimized pulse.

An extensive numerical comparison of all three RF pulses is performed including a broad set of  $B_0$  offsets and  $B_1$  scalings. The inversion efficiency is calculated for long a long repetition time (TR) with negligible  $T_1$  influence for each pair of  $B_0$  and  $B_1$  as

$$eff = -\frac{I_{(inv)}}{I_0} = -\frac{S(x) M_{z(inv)}(x) \sin \alpha(x)}{S(x) M_0(x) \sin \alpha(x)} = -\frac{M_{z(inv)}(x)}{M_0(x)} \quad (1)$$

with  $M_z$  being the z-magnetization at the end of the respective RF pulse. Furthermore,  $M_0$  is the initial magnetization and  $S(x)$  describes the signal intensity. For measurement, a slice selective excitation pulse with flip angle  $\alpha$  is necessary. This excitation pulse is affected by RF inhomogeneities as well.

In addition, all three RF pulses were investigated in vivo measurements of the knee. We used the knee coil and we set the sequence parameters repetition time (TR) and echo time (TE) to TR=8000ms and TE=2.7ms. Those experiments were performed with a fixed  $B_1$  scale of 100% and without additional  $B_0$  offset. The flip angle was set to  $90^\circ$ .

## Results

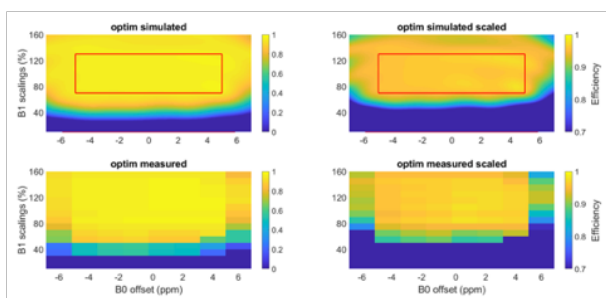


Figure 1: Simulated and measured inversion efficiencies of **optim** for  $B_1$  scaled from 0% to 160% and  $\Delta B_0$  from -7ppm to 7ppm.

Figure 1 depicts the inversion efficiency of **optim** over a broad set of  $B_0$  and  $B_1$  variations. The red box indicates the area where the optimization was done (i.e.  $B_1$  from 70% to 130% and  $B_0$  from -5ppm to 5ppm). The pulse duration was reduced to  $T_p=3.25$ ms during optimization. The top figures show the numerical efficiencies which were calculated with the relaxation times of the phantom, while the bottom figures show the efficiencies measured on the MR scanner. The plots in the left column use an efficiency scale of 0% to 100% while in the right the plots are scaled from 70% to 100%. A very good inversion efficiency of more than 94.5% can be observed within the optimized area (red box). Furthermore, the figure shows strong accordance between numerical and measurement results.

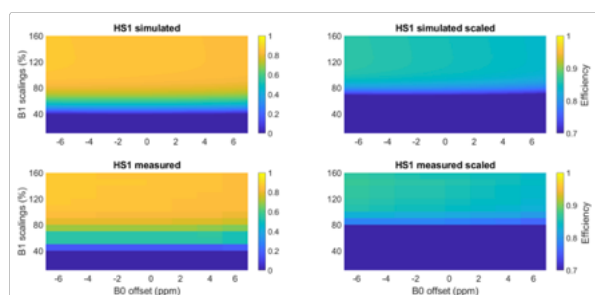


Figure 2: Simulated and measured inversion efficiencies of **HS1** for  $B_1$  scaled from 0 % to 160% and  $\Delta B_0$  from -7ppm to 7ppm.

In Figures 2 and 3 we observe the inversion efficiencies of **HS1** and **HS2**. Again, a good accordance between simulated and measured inversion efficiencies can be observed. In both cases, the efficiency of **HS1** does not reach top values. There is strong robustness among changes within  $B_0$ , but for  $B_1$  the efficiency is only acceptable for a scale of 100% and more. Below, the efficiency is less than 70%. In contrast, **HS2** shows a good inversion efficiency in the center of the plot ( $B_1$  of 100% and  $B_0$  at 0ppm). Only for a larger offset of  $B_0$  the efficiency significantly decreases.

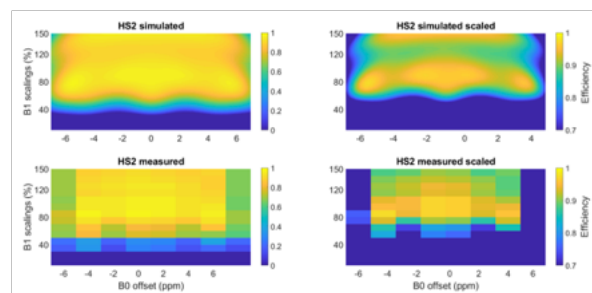


Figure 3: Simulated and measured inversion efficiencies of **HS2** for  $B_1$  scaled from 0 % to 150% and  $\Delta B_0$  from -7ppm to 7ppm.

Figure 4 displays a sagittal cross-section of the knee using no pulse in the gradient echo sequence (top) and the **optim** inversion pulse (bottom). Figure 5 depicts the inversion profile measured with **optim**, a  $B_1$ -scale of 100% and without an additional  $B_0$  offset. Between water and fat, bound protons at a resonance offset of 3.4ppm exist. Some chemical shift artifacts occur at tissue boundaries. We observe a severe decrease in signal intensity towards the proximal and distal parts in the image where the coil sensitivity and RF field strength drops to very low values. In Figure 5, the measured inversion efficiencies are depicted for all 3 RF pulses. Similarly to the phantom measurements, **optim** shows the best inversion efficiency among those 3 pulses within the defined field range. **HS1** has a decreased inversion efficiency even in the center of the knee with a fast loss in efficiency towards the coil edge. **HS2** shows a rather broad inversion capability, but with general lower inversion efficiency, in particular within the fatty bone marrow.

## Discussion

During the optimization, the pulse duration of **optim** was reduced to 3.25ms, which is substantially shorter than the long duration of 15.36ms of **HS1**. **HS2** has the same pulse duration as **optim** by design. However, the maximum amplitude is increased by 25% which makes the pulse unsuitable for many applications due to the amplitude limitations of the MR scanner. Here, optimization for **optim** was started with random initialization. If existing for the application at hand, a sophisticated initialization is in general helpful for an optimizer, and also our optimizer can be used in this classical setup. However, optimizers that robustly converge from random initialization to a competitive minimizer, are rare, and open new perspectives (e.g. finding new – possibly better - local minimizers or even quasi-global optimization by multi-random initialization).

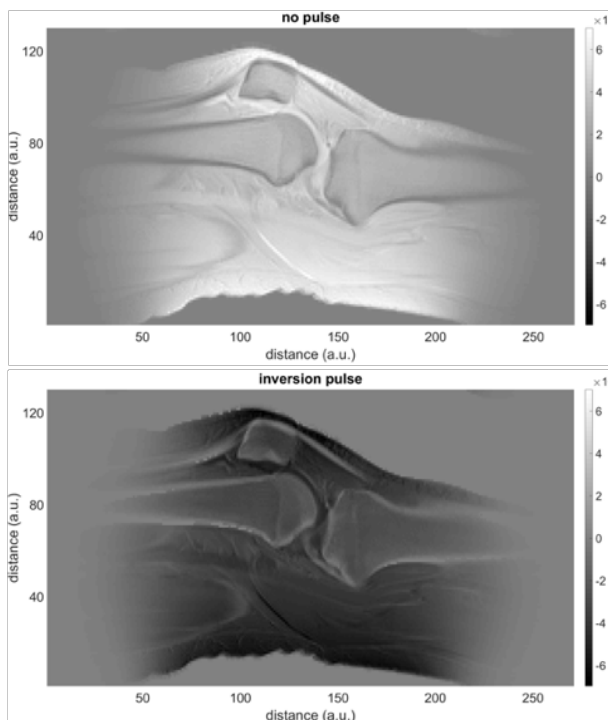


Figure 4: Sagittal cross-section of the knee. The top picture displays the image without an inversion pulse ( $S(\mathbf{x}) \cdot M_0(\mathbf{x}) \cdot \sin\alpha(\mathbf{x})$ ), the bottom picture with an inversion pulse ( $S(\mathbf{x}) \cdot M_{z(\text{inv})}(\mathbf{x}) \cdot \sin\alpha(\mathbf{x})$ ) using **optim**. The inhomogeneous signal intensity represents the inhomogeneous RF field and coil sensitivity of the knee coil.

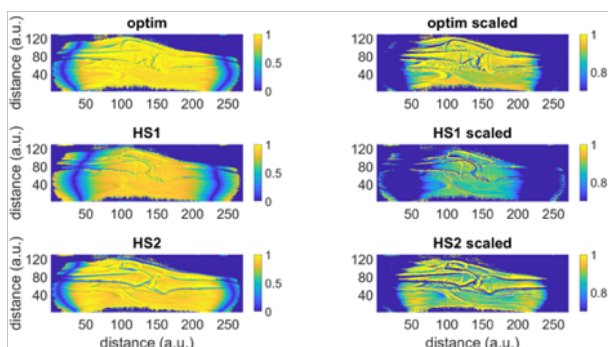


Figure 5: Inversion efficiencies for measurements of the knee. **Optim** (left), **HS1** (middle) and **HS2** (bottom). Efficiency scale of 0% to 100% (left) and 70% to 100% (right).

Figure 5 displayed the comparison of the three inversion pulses for in vivo applications. Similar to the phantom results, **optim** showed the best behaviour in terms of inversion efficiency. The long adiabatic pulse **HS1** showed already in simulation and phantom measurement only a moderate inversion efficiency which was validated in the in vivo measurement. One reason for that is its rather long pulse duration which results in relaxation effects affecting the efficiency. The short adiabatic pulse **HS2** could underline its behaviour in the in vivo experiments yielding a good efficiency. However, the main drawback is the higher amplitude

required. Furthermore, in Figure 5, we depict line artifacts due to chemical shift behaviour.

A consequent future improvement of this work would be to jointly control the slice-selective gradient for slice-selective applications. Furthermore, an extension to optimize for excitation pulses, i.e. RF pulses with a flip angle less than  $180^\circ$  would be desired.

## Conclusion

Inversion pulses were optimized within an optimal control framework with the aim of  $B_0$ - and  $B_1$ -robustness and a reduced pulse duration. The numerical and measured comparison to state-of-the-art adiabatic pulses revealed a significant improvement in terms of inversion efficiency while being short and fulfilling all physical limitations.

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