# APPLICATION OF AI TO THE 1D VENTILATION ANALYSIS OF A 43KM COMPLEX ROAD TUNNEL NETWORK: MADRID CALLE30

<sup>1</sup>Juan Manuel Sanz, <sup>2</sup>Fabián De Kluijver, <sup>2</sup>Alberto López, <sup>3</sup>Javier Berges, <sup>3</sup>Mar Martinez, <sup>1</sup>Guillem Peris <sup>1</sup>Sener, ES <sup>2</sup>JVVA, ES <sup>3</sup>Madrid Calle 30, ES

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## ABSTRACT

Madrid Calle 30 ring road tunnels are a very complex road tunnel network, with a total extension of 43 km opened in 2007. In order to update the ventilation control algorithms, it has been necessary to analyze the records of all sensors and ventilation equipment since inauguration with the objective of characterize the current capacity of the system, the influence of the different parameters and learn from the analysis of past events. Due to the extension and complexity of the network, and the huge amount of field data available, an Artificial Intelligence (AI) system (Respira®, from SENER) has been used to evaluate the current performance of the system.

In parallel, a 1D simulation model of the whole ventilation system (as a digital twin) has been generated. The uncertainty in some parameters of the 1D model, and its size and complexity, has driven the design team to automatize its calibration. A surrogate model of the 1D model was built on Python by training it with a sample of 3,000 1D simulations, where uncertain parameters were modified randomly within ranges given by expert knowledge. Differential Evolution method is used for calibration [1], obtaining a set of parameters that minimizes an error function between the model prediction and the field data.

With this calibrated digital twin, it has been possible to optimize the ventilation algorithms for the different events foreseeable in the different areas of the M30.

Keywords: Algorithms, ventilation, Artificial Intelligence, simulations.

# 1. INTRODUCTION

The tunnels of Madrid Calle30 form a complex road network over 43 km long, with 938 fans, 85 ventilation shafts, 430 anemometers, and 284 gas detection sensors. Inaugurated in 2007, its ventilation control system is being refurbished as the control equipment is reaching the end of its life cycle. In addition to this, the operation of the ventilation system is being analysed and updated, taking into consideration the experience acquired in these years, the current situation of the infrastructure and the capabilities of the new control system.

Updating the ventilation algorithms requires adequate knowledge of the system's capabilities and behavior, as well as a digital model that allows knowing in advance the system's responses to different events or future actions. It is vitally important that the digital model is properly calibrated so that the results obtained accurately predict the behavior that the ventilation will actually have in the tunnel. Once the capacity and configuration of the ventilation system is adequately known and a properly calibrated simulation model is available, the development of the ventilation algorithms is similar to that of any other tunnel. At this point the difference is only the large number of zones and equipment that must be considered for operation and fire situations.

For the analysis of ventilation and historical records of such a complex, extensive network with so much data, the application of artificial intelligence tools is a vital help. The application of automatic tools that allow comparison of the results of the simulations with those recorded during normal operation or during the various ventilation tests developed is also of great help. It is necessary to take into account the high number of parameters to be considered in the different areas of the infrastructure, in relation to circulation and ventilation itself.

# 2. DESCRIPTION OF THE INFRASTRUCTURE

The M-30 is the busiest road in Spain: it registers a daily average of 1.3 million trips (vehicles), with peaks of one and a half million journeys in 24 hours. We are talking about 475 million vehicles and 570 million people (users) per year. It is the main ring road of the city of Madrid with a mixed configuration of tunnels and open-air road.

The M-30 is the most extensive network of urban tunnels in Europe (48 kilometers of tunnels, equivalent to an underground lane of 118 kilometers. The tunnels have multiple entrances (21) and exits (22), as well as a wide variety of configurations and number of lanes in their different areas.



Figure 1: Scheme of the M30 tunnels

It also has different ventilation modes in its different areas (longitudinal, longitudinal with specific extractions, semi-transverse and transverse). The complexity of the tunnel network and its ventilation can be seen with some of the figures of its equipment:

- 430 anemometers (347 inside the road tunnels, rest in auxiliary tunnels and portals)
- 75 CO sensors
- 68 NOx sensors
- 141 opacimeters
- 521 jet fans
- 127 smoke extraction shafts for support longitudinal ventilation (normally each of then has two axial fans)
- 85 ventilation shafts with 163 axial fans (part of them are reversible)

# 3. NALYSING FIELD DATA RECORDS WITH ARTIFICIAL INTELLIGENCE

# 3.1. VARIABLES

To analyze the current capacity of ventilation and the influence of the different variables on this system, the records of all relevant elements from the last 12 years (2011-2022) were extracted from the control center. These records included:

- Indoor and outdoor environmental sensors (air velocity, wind, temperature, CO, NO and NO2, Opacity, etc.)
- Activation of ventilation equipment (jet fans, ventilation shafts, punctual extractions, air filters, dampers information, etc.)
- Traffic data
- Fire alarms and emergency exit sensors.
- Auxiliary variables of the control system

The information collected exceeds 3 TB of database files. Due to the high number of elements for which information is available, the way in which the data is recorded, the great diversity of variables considered and the different scales of the variables, it is practically impossible to process all the records manually or with conventional tools. From the tender stage, it was proposed to process all this information using artificial intelligence.

## 3.2. PROCESSES, ANALYSIS, REPRESENTATION AND MINING OF DATA

Once all the information received was registered, it was processed and filed homogeneously in a cloud database. In addition, an information viewer was developed with multiple possibilities for filtering and selecting variables, zones and times. This has been of great help to engineers to simultaneously analyze different variables and corroborate the correlations provided by artificial intelligence. It has also allowed us to analyze various specific events that the AI discards because they are not frequent, such as fire drills or real fires.



Figure 2: Example of the information viewer. Average air velocity from tunnel anemometers in the period selected.

Once all the information had been stored, the data was processed, filtering all the information that corresponds to specific events or registration failures that do not fit the main correlations that govern the ventilation of the tunnel. This guarantees that the data being analyzed is of quality for the purpose of the study.

To facilitate the analysis of the behavior of the ventilation of the infrastructure, AI techniques have been used to identify the equipment of greatest interest. With this, a large part of the equipment has been grouped into clusters that have similar behaviors or patterns. This allows engineers to see areas where ventilation behavior is similar, as well as greatly reducing the search for physical correlations between events or variables. These clusters have also helped to quickly locate sensors whose measurement is not sufficiently precise (or even incorrect) and have allowed the operator to know which equipment needed to be checked or adjusted. Measurements from devices that belong to clusters with inaccurate measurements are removed from subsequent data analysis and the model calibration process.



Figure 3: Cluster of anemometers with similar behavior in a given month.

The analysis of traffic data also allows us to obtain correlations in the different areas of the road between traffic intensity and speed and air velocity to analyze the piston effects in the different zones.



Figure 4: Correlation between traffic intensity and speed in the different areas of the tunnel.

Another analysis carried out has been the distribution of temperatures along the tunnel, its variation with time and its relationship to the outside temperatures and air speed. This has allowed us to know its influence on the natural draught during the normal operation of the tunnels. In general, it has been proven that the thermal effects in normal operation are much lower than those of the piston effects. During low traffic hours, thermal effects tend to exhibit repetitive patterns since they occur at the same times every day under very similar thermal conditions. Due to the large air currents in the tunnels caused by traffic, the tunnel temperature quickly adapts to the outside diurnal average. In Madrid, at night, the temperature is always lower than the daily average, so every night the tunnel temperature is higher than the outside temperature and the natural drafts are always the same. Likewise, it is observed that

temperatures tend to be colder in the West area that in the East (TBM tunnel), partly due to the presence of the Manzanares River and partly due to the greater number of connections with the outside of this area.



Figure 5: Average temperatures in the different areas of the tunnel in 2021.

Additionally, other artificial intelligence tools developed by Sener have been applied to the data using the Respira® tunnel ventilation control solution that has allowed, among other things, analyze the following:

- Analysis of hours in which the contaminant values are located at the different control thresholds.
- Relationship between traffic intensity and gas concentration.
- Establish homogeneous areas of the tunnels in relation to the different variables of interest.
- Selection of variables that affect tunnel ventilation and categorization in relation to their influence on ventilation.
- Detection of variables whose effect on ventilation overlaps with those of others already considered.

# **3.3. RESULTS OBTAINED WHEN APPLYING AI**

The methodology used by Sener in their AI product Respira® for control the ventilation in tunnels is the next:



Figure 6: Methodology used by Sener in the tunnel ventilation control system applying IA.

In this project we have used the steps 2, 3 and 4. The most notable results are the following:

- Categorization of the variables that affect ventilation, making it easier to focus on the most important ones for the calibration of the simulation model and mainly discard those whose influence is minimal.
- Provide important tools for visualization and analysis of ventilation in different areas and events.
- Grouping of the different variables into clusters to reduce the number of situations to be analyzed manually.
- Quick obtaining of relevant values in common or specific situations or application of formulas considering the values over time of different variables.
- Detection of equipment with low precision in its measurements or with error.
- Elimination of incorrect or non-representative values of system operation.
- First analysis of the efficiency of the different jet fans and ventilation shafts.
- Obtaining certain correlations between variables (for example piston effect) or interpretation of repetitive events or direct physical relationship (temperature distribution, thermal effects, ...).

Additionally, it has been verified that all the information and tools are available to predict the behavior of the ventilation of the entire infrastructure and carry out control directly through AI assuring algorithms calibrated with real focus on safety and energy efficiency.

# 4. SIMULATION MODEL GENERATION AND CALIBRATION

The simulation model considered for the ventilation of the tunnel network is a 1-D model. This type of models apply to systems in which longitudinal dimensions are much bigger than transverse dimensions. For this reason, flow variables can be represented varying only along longitudinal coordinates not in the transverse directions. The value of each variable in each point represents the average value of that given variable in the transverse section of the corresponding point. The software used for the 1-D model is IDA-Tunnel, which has been validated for railway and for road tunnel networks. This model can be used to study both normal and fire ventilation scenarios in the tunnel network.

As in any physical model, there are some uncertainties in the definition of the inputs. In order to find values of these inputs that provides a good representation of the actual behavior of the system, a calibration process has been performed. In a first step, the tunnel was modeled without considering the ventilation (what we will call "physical model"); once the geometry and physical parameters of the tunnels were adjusted, the parameters of the different fans and shafts were adjusted, to obtain the "ventilation model" or "digital twin".

In the input set (input vector) of the physical model, two types of inputs have been considered:

- Model parameters ( $\theta$ ): these are values of inputs of the physical model that are constant in time and have an associated uncertainty in its value.
- State inputs (*x*): these values are inputs of the model that are variable with time. In the case of the model considered, these inputs are the traffic flows in each of the tunnel segments and the ambient temperature. These inputs are assumed to be measured and part of the field data. For the purpose of the presented study, no uncertainties are considered for these inputs.

The calibration of the model parameters consists of finding the combination of those model parameters ( $\theta$ ) that provides the best fit between the output of the 1D simulation and the field data for different state conditions (traffic vector and ambient temperature). The output variable

that has been used for calibration is the air velocity in the tunnel, measured in each of the anemometers of the infrastructure. For the calibration process, the model parameter space needed to be explored by comparing the outputs of the simulations with the field data for different combinations of the model parameters and state inputs. As will be exposed below, the complexity of the model leads to a great number of parameters to calibrate. With such parameter vector, exploring the parameter space by directly using the simulation model would result in an unaffordable computational time.

In order to solve the computation time issue in the calibration process, a surrogate model of the simulation model has been defined. The surrogate model is a mathematical model that aims to provide (for a given set of inputs) equivalent outputs to the outputs provided by the 1D simulation model, but with a much lower computation time. Once the surrogate model is defined, it can be used in substitution of the simulation model during the calibration process.

Once the physical model has been calibrated, the ventilation is calibrated by adjusting the parameters and variables directly related to the activation of jet fans and shafts. To do this, it is verified that the variation in air velocity over time is accurately reproduced when the different equipment is activated. For this purpose, various tests were carried out in different areas of the tunnels. The complete model (digital twin) was considered adequately calibrated when the simulation results accurately reproduced the measurements of the different tests and the recorded field's data.

# 4.1. MODEL BUILDING

In order to build the 1D simulation model, information has been collected from the different project documentation of the infrastructure. The tunnel network uses different ventilation strategies depending on the part of the network considered, with some areas using longitudinal ventilation and others using transverse or semi-transverse ventilation.

In Figure 7, a diagram of the 1D simulation model is represented. The model is composed by 194 tunnel segments. These elements are defined between two air flow bifurcations that can be tunnel bifurcations, connection to ventilation shafts or portals. In order to study situations without mechanical ventilation (used for example in the calibration process), tunnel segments are grouped taking into account only road bifurcations, resulting in 100 groups of tunnel segments.



Figure 7: Diagram of the 1D simulation model.

# 4.2. CALIBRATION OF THE "PHYSICAL MODEL"

# 4.2.1. PARAMETERS TO CALIBRATE

As it was mentioned above, in any physical model there are some uncertainties in the definition of its inputs.

In the tunnel network considered there are many different vehicle types driving through it. For this reason, there is a considerable uncertainty in the definition of those parameters related to the aerodynamic forces of the traffic (piston effect in the tunnel). In the case studied, these parameters are the front area and the drag coefficient of the vehicles.

The model does not include local geometry changes that generate pressure losses in the tunnel (lay-bys, local height changes, beams, etc.), given the impossibility of including them in the model due to their quantity and complexity. These local pressure losses will be absorbed by the friction coefficient during the calibration process. For the friction coefficient in the model, a unique representative value  $\lambda^k$  is defined for each group of tunnel segments k.

Similarly, there are numerous cross-sectional dimensions for a given tunnel segment, being an unaffordable task to include all of them in the model. For that reason, in the simulation model these have been simplified. Since the tunnel cross sections are approximately rectangular, the simplification used considers a variable width depending only on the number of lanes, but a constant height for each group of tunnel segments. These heights will be also calibrated obtaining a representative value for the whole segments in a group.

The parameters used in the calibration process have been selected with a tradeoff in representing accurately the behavior of the infrastructure, but not increasing the number of parameters in an unreasonable way. In some cases, the parameters selected are not directly inputs of the model, but these inputs need to be calculated from these ones. This is the case of the friction coefficients and the transverse section dimensions.

During the calibration, a steady state approach has been used. Due to this, the tunnel wall temperature evolution is not simulated during the calibration process. To reduce the error due to this effect, an additional parameter  $(a_{temp})$  has been introduced which allows subsequent adjustments of the model for the events in which it intervenes.

For the calibration process, the limits for the parameters need to be defined. In order to obtain realistic calibration results, expert criteria based in experience studying ventilation of tunnel networks is used. In Table 1, the number of parameters and their limits are summarized. The vector of parameters  $\theta$ , considers the 202 parameters represented in the table.

Parameter	Number of parameters	Minimum value	Maximum value
Vehicle aerodynamic drag coefficient $(c_x)$	1	0.3	0.7
Vehicle front area coefficient $(A_f)$	1	1.9	3.3
Global friction coefficient $(\lambda_{global})$	1	0.01	0.035
Local friction coefficient $(\lambda_{local}^k)$	100	0	0.035
Tunnel heights $(h^k)$	98	Different limits depending on the group k	
Wall temperature parameter $(a_{temp})$	1	0	1
TOTAL	202		

Table 1: Parameters to calibrate.

## 4.2.2. SURROGATE MODEL

As it was mentioned above, the surrogate model aims to provide equivalent outputs to those provided by the 1D simulation model, but with a shorter computation time. The surrogate model that was built represents the air velocity in each of the anemometers, corresponding to a steady state simulation with a constant traffic profile, constant ambient temperature, and without mechanical ventilation. The input set of the surrogate model includes the vector of the parameters to calibrate ( $\theta$ ) and the vector of state inputs (x).

The type of state of the system considered for the calibration is a steady state situation without mechanical ventilation, which is assumed to be reached with a constant traffic flow and ambient temperature for one hour (period that is considered representative and in accordance with the traffic field data available). Therefore, hours of the year in which some of the ventilation was activated (jet-fans or ventilation shafts) have been filtered out and not considered for calibration.

Starting from a complete year of field data (those from the last full year available, which is 2021), after ventilation has been filtered, 3884 hours of data remains to perform the calibration. Looking for the state input, although there are not two hours with the same state input vector (x), the state of the system is repeated periodically along the year in an approximate way, resulting in some groups of state input vectors that are very similar (and would induce similar velocity in the tunnel). To take advantage of this, and to further reduce the computation time, 500 clusters have been defined (regarding the state input data), grouping each of the hours of the year 2021 in one of those clusters. For each cluster of hours *j*, an average state input vector  $x_j$  has been defined.

Surrogate model used has been built by using a gradient boost decision tree regression model [2] [3]. The regression model is fitted with a sample of 3000 simulations performed with the 1D model, varying the input vector  $(x, \theta)$  in each of the simulations. Input for all these simulations have been generated with a Latin hypercube sampling method (LHS) with the range of variation of inputs limited to the same range that will be considered for calibration (Table 1).

With this set of 3000 simulations, the output of each of the 303 anemometers considered could be evaluated, fitting a surrogate model for each of these anemometers. The complete surrogate model results, therefore, in the combination of these 303 surrogate models.

Performance of the predictions provided by the surrogate model compared with the velocity outputs provided by the 1D model are represented in form of a  $R^2$  score for each of the anemometers. In Figure 8, histogram of the  $R^2$  scores is represented. As it is possible to see in this figure, the surrogate model provides a good prediction of the results of simulations.



Figure 8: Left - General performance of surrogate model represented as a histogram of R2 scores of the surrogate models of each of the anemometers. Right - Example of performance of surrogate model of two of the anemometers considered. Predictions of surrogate models compared against result of the simulations.

## 4.2.3. CALIBRATION OF THE MODEL PARAMETERS

Since the surrogate model has demonstrated to represent with enough accuracy the simulation model (for the conditions of interest) but with a much lower computation time, for the purpose of the calibration process the outputs of the surrogate model have been used substituting the role of the simulation model. With this strategy, an affordable computation time to explore the model parameter space is achieved.

For the field data corresponding to the air velocity measured by each of the anemometers, raw data is considered. This raw data contains measurements of air velocity that are not uniformly distributed in time, being the number of measurements for certain anemometer and certain hour of the year very variable. Measurements are classified in the cluster corresponding to the hour when the measure was taken. Pairs of cluster-anemometer with a low number of measurements (less than 25) are considered not representative enough and have been eliminated from the calibration process. From these filtering there are some anemometers that result with no valid clusters, reducing the number of anemometers usable for calibration from 303 to 289.

For a given value of the parameter vector  $\theta$ , error functions are defined to measure the difference between simulation results and field data. An error function  $e_i(\theta)$  is defined for each anemometer, and from those values a global error function  $e(\theta)$  is defined. Equations used in the definition of these error functions are summarized in Figure 9.



Figure 9: Definition of errors between field data and model results.

The objective of the calibration process is to find the value of  $\theta$ , that minimizes the global error function. The surrogate model allows to quickly iterate through  $\theta$  calculating in each step  $y_{ij}^{sim}(\theta)$  for all the clusters and anemometers and obtain the corresponding value of  $e(\theta)$ . Differential evolution algorithm has been used to explore the parameter space finding the value  $\theta$  that minimizes the value of the error  $e(\theta)$ .

In **Figure 10** the effect of the calibration on the velocities predicted by the model is represented. In this figure, the air velocity of different anemometers is represented against the value of traffic flow in the tunnel segment where the anemometer is placed. Three data sets are represented for each anemometer, field data (blue dots), outputs of surrogate model before calibration (red dots), and outputs of surrogate model after calibration (green dots).

The field data shows a greater amplitude than those generated by the simulation model, for 2 main reasons: greater variability in the environmental conditions (punctual wind, positive or negative natural draught due to sudden changes in temperature, etc.); and turbulence recorded by the anemometer (three-dimensional effects). It can be seen in the figure, that the calibration of the model improves the prediction of the air speed value and its relation to traffic (piston effect).

In **Figure 11** histograms of the distribution (by anemometer) of errors of the surrogate model in the prediction of field data are represented. Two histograms are represented, by using the parameter vector  $\theta$  before and after calibration. The figure shows the improvement in the predictions with the calibrated values of the parameters.



Figure 10: Air velocity (m/s) for different anemometers against the traffic flow (vehicles/hour) in the tunnel segment where the anemometer is placed. Three data sets are represented for each anemometer, field data (blue dots), outputs of surrogate model before calibration (red dots), and outputs of surrogate model after calibration (green dots).



Figure 11: Histograms of the distribution of errors of the surrogate model in the prediction of field data. Left – Before calibration. Right – After calibration.

# 4.3. CALIBRATION OF THE VENTILATION SYSTEM MODELLING

# 4.3.1. FIELD AERODYNAMIC TESTS

With the information from the control center records and the equipment data, the necessary parameters of the ventilation system were available, but some uncertainties remained for the simulation model. To minimize these uncertainties, a series of field tests were defined and conducted.

The first round of tests consisted of measuring the air flow of each of the shafts (85) for their different operating regimes (different operating speeds of their axial fans) independently or in combination with nearby ventilation equipment (jet fans or ventilation shafts). From these tests, the most important data has been the current maximum air flow of each shaft. Additionally, first approximations of the efficiency of the jet fans and the influence of each shaft on the adjacent sections were obtained.

Due to the frequency of data recording from the control center, analysis of transient processes when switching ventilation equipment on or off was not possible. For this purpose, 30 aerodynamic tests were defined and carried out. In these tests, 2 anemometer arrays were placed at different points in the tunnel (normally on both sides of a shaft) and 1 additional array in the shaft. Various shaft regimes and nearby equipment (jet fans, shafts and specific extractions) were turned on, obtaining variations in air flow and velocity throughout the tests. It also allowed knowing the air flow in both directions for the reversible shafts.



Figure 12: Distribution and measurements of anemometers during aerodynamic testing (left in shaft, right in tunnel).

# 4.3.2. CALIBRATION OF VENTILATION SYSTEM

Once the model has been calibrated using field data in states without ventilation of 2021, the aerodynamic tests performed in the infrastructure were simulated. With the simulations of these tests, it is verified that the model is able to represent the behavior of the system in the different ventilation regimes tested. In this model, calibration was only necessary to slightly adjust some parameters related to the efficiency of the ventilation equipment, shaft air flow or wall temperature (natural air draught). In Figure 13 comparison of the simulation results and the experimental data from one of the aerodynamic tests is represented.



Figure 13. Results of air velocity results obtained in one of the aerodynamic tests. Comparison of simulation results (10NC62AI01) with the measurements of the six anemometers used in the test (T1-1, T2-1, T3-1, T1-2, T2-v, T3-2).

# 4.4. APPLICATION OF THE MODEL TO THE DEFINITION OF ALGORITHMS

Once the model has been calibrated, the possible ventilation actions required for sanitary and fire ventilation during different events have been simulated. This way, the jet fans and shafts (with their specific regimes) that must be active at all times have been determined, along with the time periods and transitions required for the system to adapt to the potential evolution of the event.

This has made it possible to define in detail the sanitary and fire ventilation algorithms for each position of the infrastructure.

In the next stages of the work, once the ventilation algorithms have been implemented in the control center, it will be verified with aerodynamic and smoke tests that the system response adjusts to what was predicted by the calibrated model.

With the model it has also been possible to analyse the equipment that has the greatest efficiency to achieve the objectives set for each point of the infrastructure, as well as to optimize the exact number of equipment to be turned on and the activation regime required for the ventilation shafts.

# 5. SUMMARY AND CONCLUSION

The use of new technologies as AI enables engineers to process and analyze a huge amount of information. Madrid Calle 30 Tunnels are a very complex road tunnel network of 43 km inaugurated in 2007. More than 3TB of information related to the ventilation system has been processed and analyzed by an AI system called Respira®. The use of cloud databases and artificial intelligence tools has been demonstrated to be essential in order to characterize the capacity and operation of tunnel ventilation as well as the analysis of certain events of great interest.

With all the information processed, a digital twin of the ventilation system has been created. Once calibrated, it enables the analysis of the optimal response to each event and the definition of the ideal algorithms for both normal operation and fire events.

The calibration process of the digital model has required, given its complexity and extensive recording of verification data, the use of computer tools and artificial intelligence techniques to automate the processes. It should be noted that the calibrated model reproduces with great precision all the field tests carried out both in their global values and their evolution over time (stationary and transient processes).

All this analysis would not be possible without the data collected from the control center since 2011, which remarks the importance of store the information of the facilities of the tunnel through the SCADA system.

#### 6. REFERENCES

- [1] Scipy optimize differential evolution SciPy v1.12.0 Manual. (n.d.). https://docs.scipy.org/doc/scipy/reference/generated/scipy.optimize.differential\_evolution.ht ml
- [2] Scikit-learn: Machine Learning in Python, Pedregosa et al., JMLR 12, pp. 2825-2830, 2011
- [3] 1.11. Ensembles: Gradient boosting, random forests, bagging, voting, stacking. (n.d.). Scikitlearn. https://scikit-learn.org/stable/modules/ensemble.html#gradient-boosting